THE WEB CONFERENCE ACM

The Grainger College of Engineering
IBM-Illinois Discovery Accelerator Institute

AIFARMS
Artificial Intelligence for Future Agricultural Resilience, Management, and Sustainability

NSF

Neural Contextual Bandits for Personalized Recommendation

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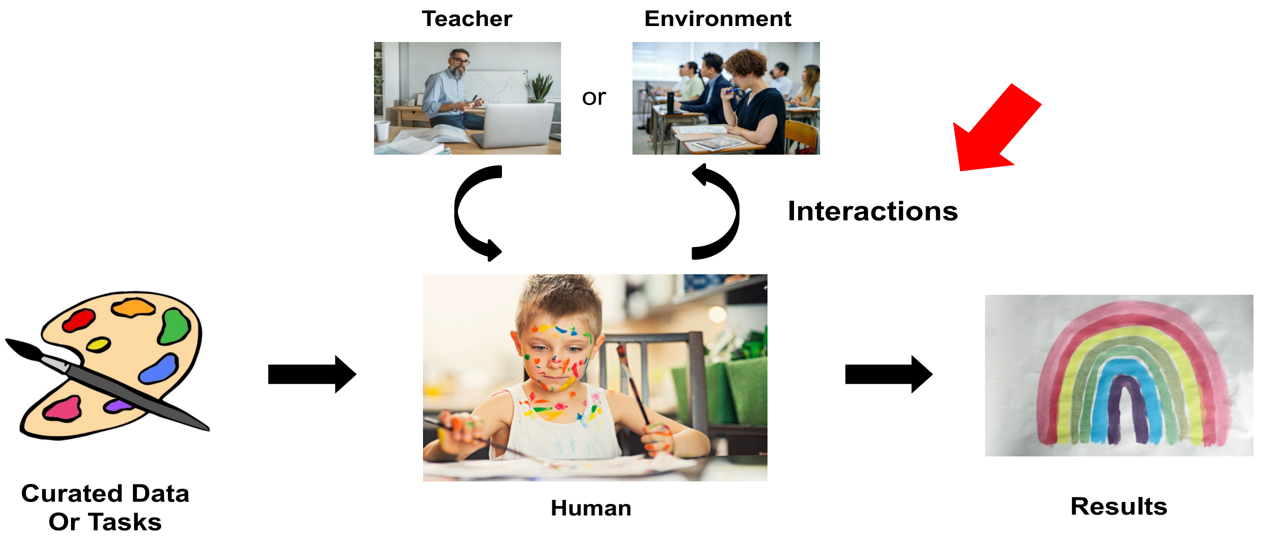
Time: 9:00 AM – 12:30 PM, 13 May 2024

Location: Virgo 1, Resorts World Sentosa Convention Centre, Singapore

Website: www.banyikun.com/wwwtutorial/

ILLINOIS

Interactions in Machine Learning



Teacher

or

Environment

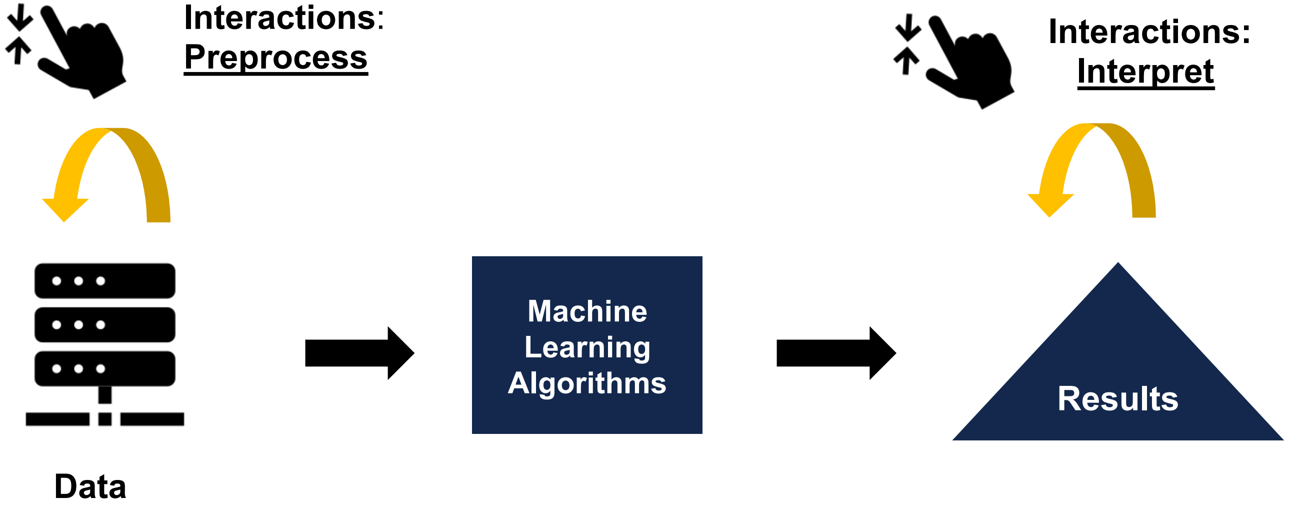
Interactions

Curated Data
Or Tasks

Human

Results

Interactive Learning
(Human)



Interactions: <u>Preprocess</u>

Interactions: <u>Interpret</u>

Data

Machine Learning Algorithms

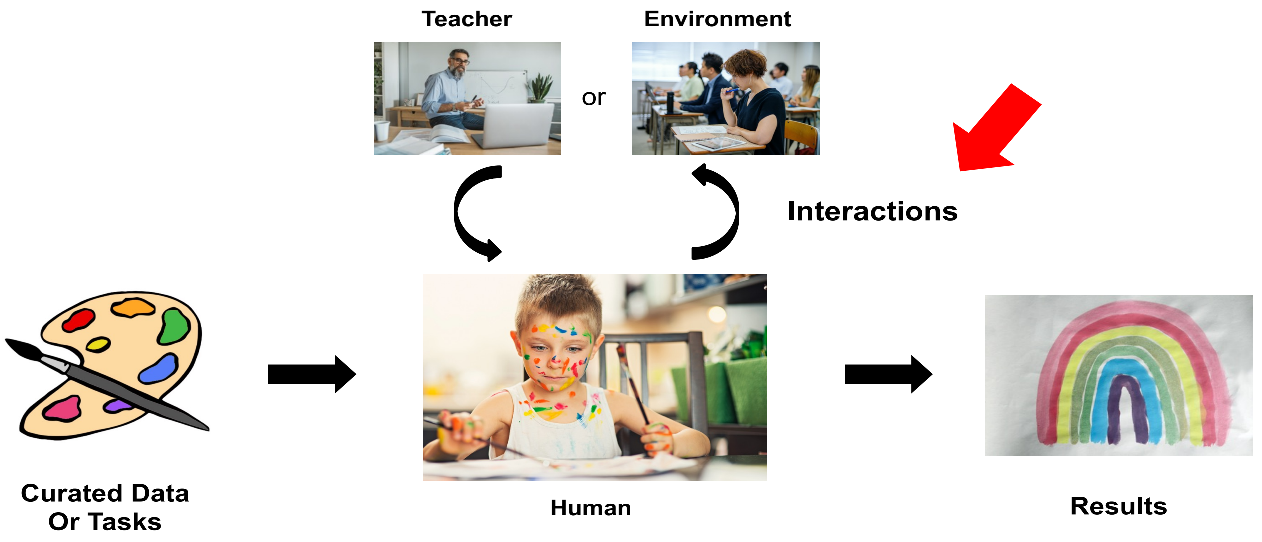
Results

Machine Learning
(Conventional)

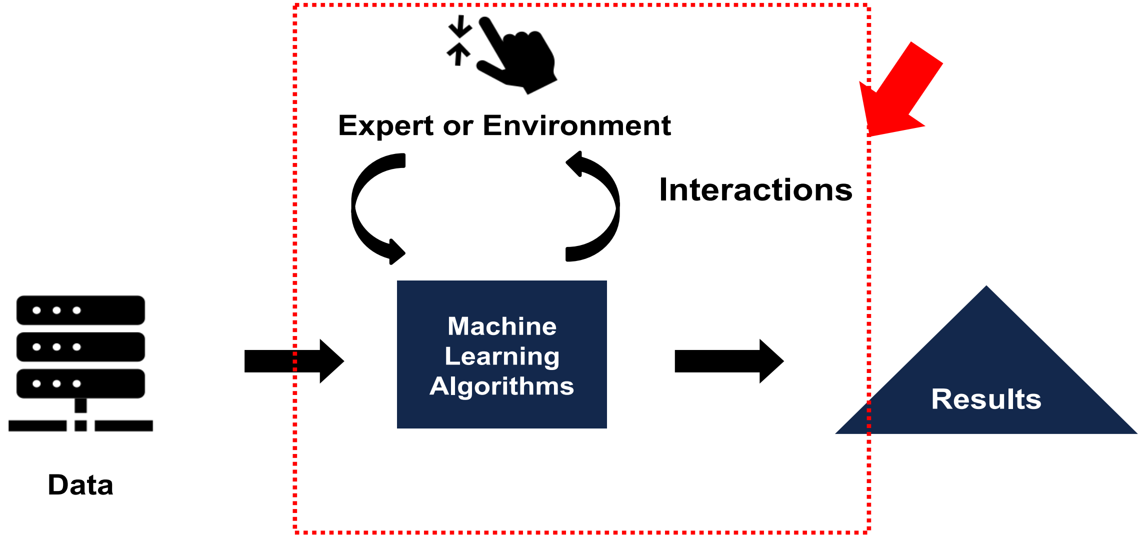
☹

1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.
2. Fails, Jerry Alan, and Dan R. Olsen Jr. "Interactive machine learning." *Proceedings of the 8th international conference on Intelligent user interfaces*. 2003.
3. Teso, Stefano, and Kristian Kersting. "Explanatory interactive machine learning." *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*. 2019.

Interactions in Machine Learning



Interactive Learning (Human)



Interactive Machine Learning

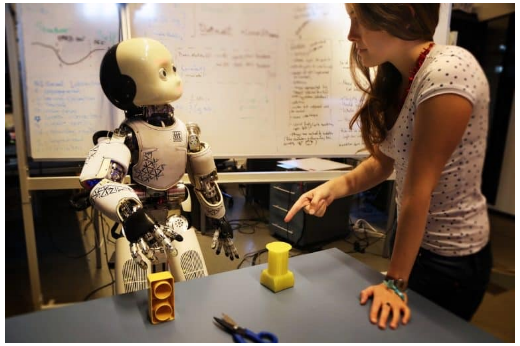
☺

1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.
2. Fails, Jerry Alan, and Dan R. Olsen Jr. "Interactive machine learning." *Proceedings of the 8th international conference on Intelligent user interfaces*. 2003.
3. Teso, Stefano, and Kristian Kersting. "Explanatory interactive machine learning." *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*. 2019.

Interactive Machine Learning and Applications

- **Interactive Machine Learning (IML)** is the core of **Artificial Intelligence (AI)**.

(1) **Recommender Systems**



(2) **Robot Learning**

(3) **Language Model**

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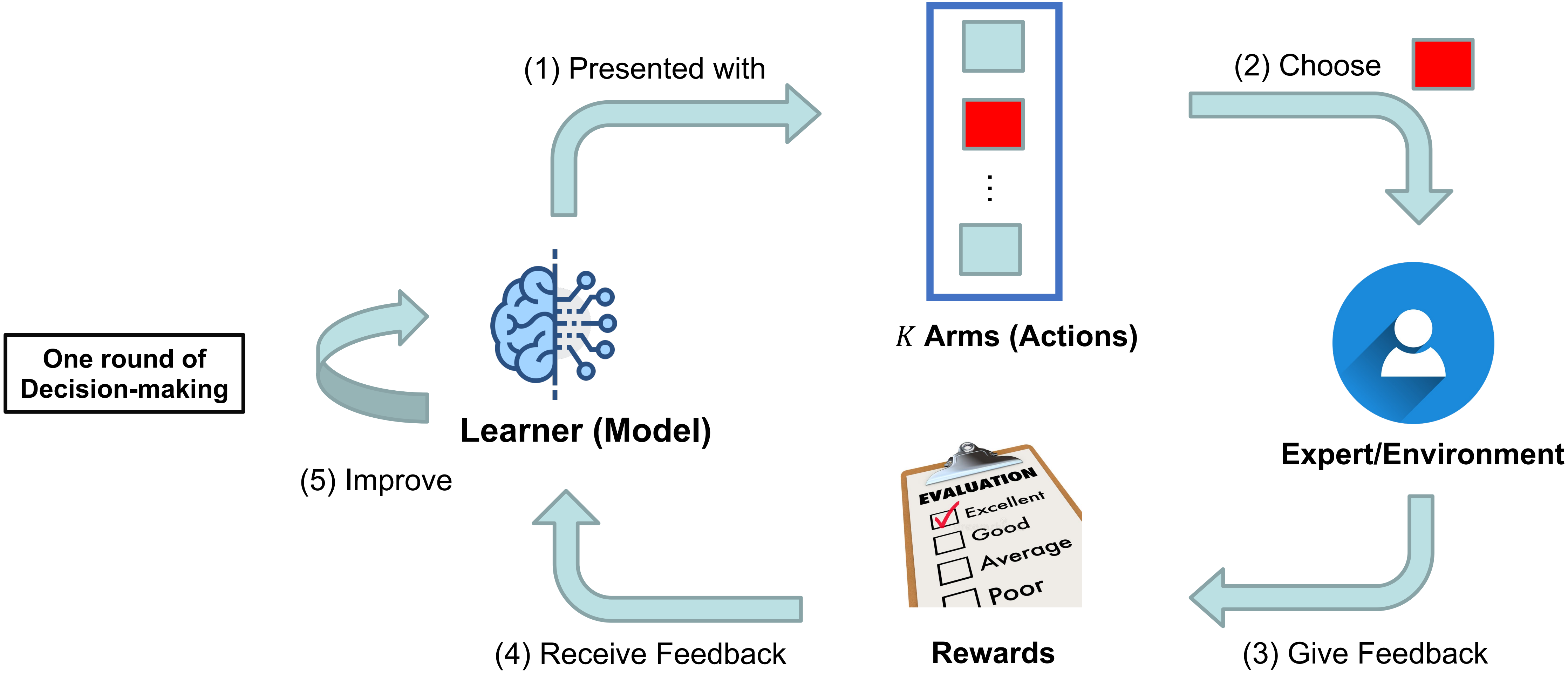
1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.
2. Fails, Jerry Alan, and Dan R. Olsen Jr. "Interactive machine learning." *Proceedings of the 8th international conference on Intelligent user interfaces*. 2003.
3. Teso, Stefano, and Kristian Kersting. "Explanatory interactive machine learning." *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*. 2019.

Sequential Decision-Making: Bandits Formulation

- Many IML scenarios can be formulated as **sequential decision-making**.



One round of Decision-making

(1) Presented with

(2) Choose 🟥

K Arms (Actions)

Learner (Model)

Expert/Environment

(5) Improve

(4) Receive Feedback

Rewards

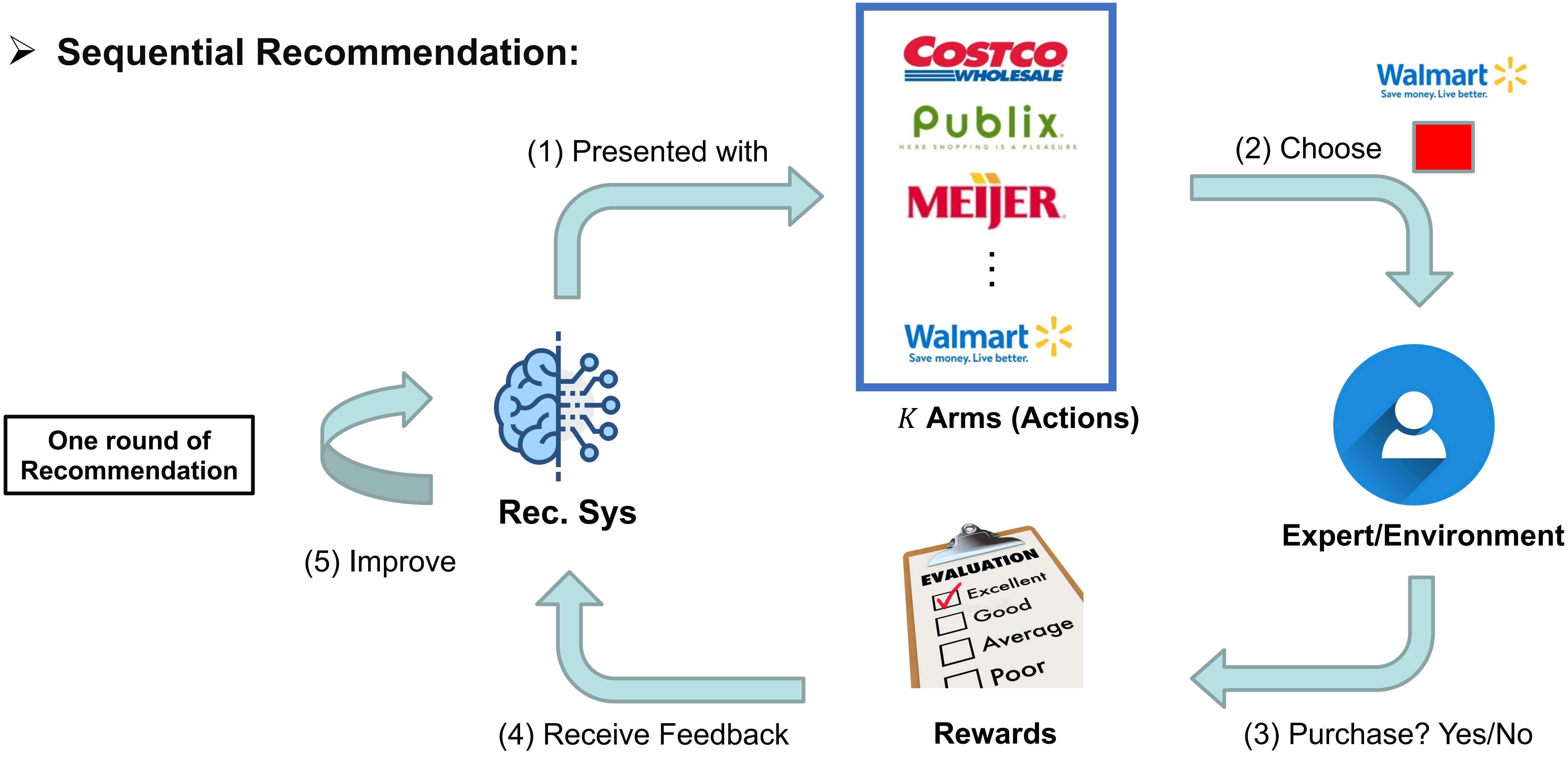
(3) Give Feedback

---

1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.
2. Fails, Jerry Alan, and Dan R. Olsen Jr. "Interactive machine learning." *Proceedings of the 8th international conference on Intelligent user interfaces*. 2003.
3. Teso, Stefano, and Kristian Kersting. "Explanatory interactive machine learning." *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*. 2019.

Sequential Recommendation: Bandits Formulation

- **Sequential Recommendation:**



(1) Presented with

K **Arms (Actions)**

(2) Choose 🟥

Expert/Environment

(3) Purchase? Yes/No

Rewards

(4) Receive Feedback

Rec. Sys

(5) Improve

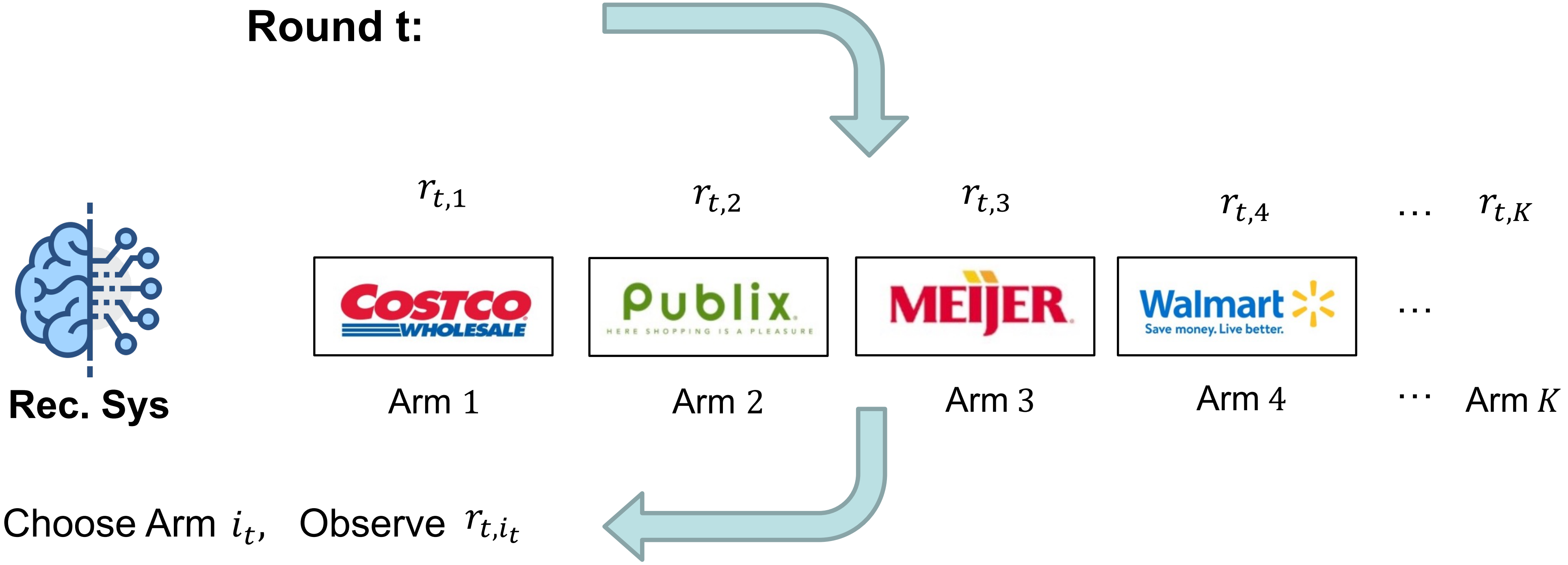
One round of Recommendation

---

1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.
2. Fails, Jerry Alan, and Dan R. Olsen Jr. "Interactive machine learning." *Proceedings of the 8th international conference on Intelligent user interfaces*. 2003.
3. Teso, Stefano, and Kristian Kersting. "Explanatory interactive machine learning." *Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society*. 2019.

Sequential Recommendation: Objective

Goal: Maximize $\sum_{t=1}^{T} \mathbb{E}[r_{t,i_t}]$ **Or** Minimize $\sum_{t=1}^{T} (\mathbb{E}[r_{t,i_t^*}] - \mathbb{E}[r_{t,i_t}])$, where $i_t^* = \arg\max_{i \in [K]} \mathbb{E}[r_{t,i}]$.



Round t:

$r_{t,1}$ $r_{t,2}$ $r_{t,3}$ $r_{t,4}$ $\cdots$ $r_{t,K}$

Rec. Sys

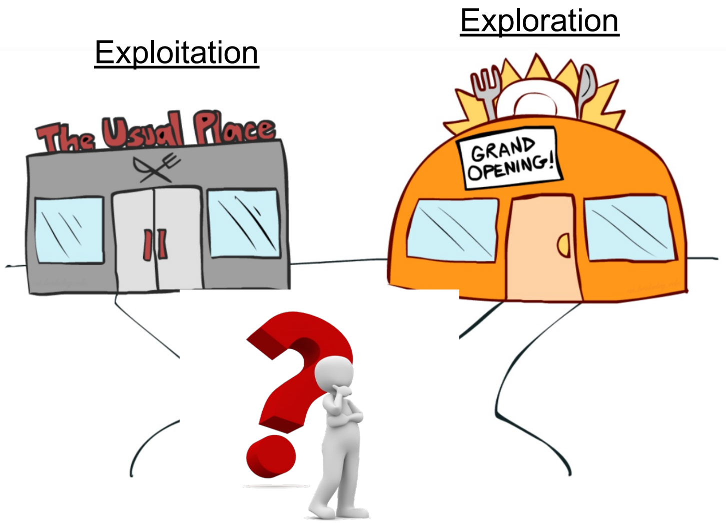
Arm 1 Arm 2 Arm 3 Arm 4 $\cdots$ Arm K

Choose Arm i_t, Observe r_{t,i_t}

1. Ernst, Damien, and Arthur Louette. "Introduction to reinforcement learning." 2024.

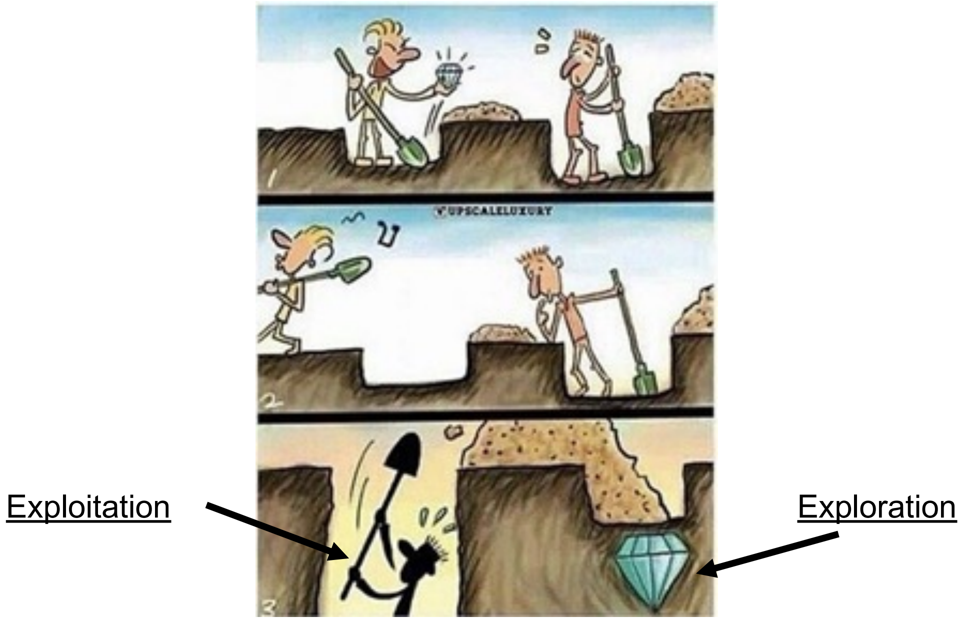
Exploitation VS Exploration in Sequential Decision-Making

- Dilemma of **exploitation** and **exploration** is ubiquitous in **human decision-making**.



(1)

Exploitation:
Exploit past data or observations.
E.g., estimation by a greedy model



(2)

Exploration:
Explore new knowledge for long-term benefit.
E.g., take uncertain actions

VS

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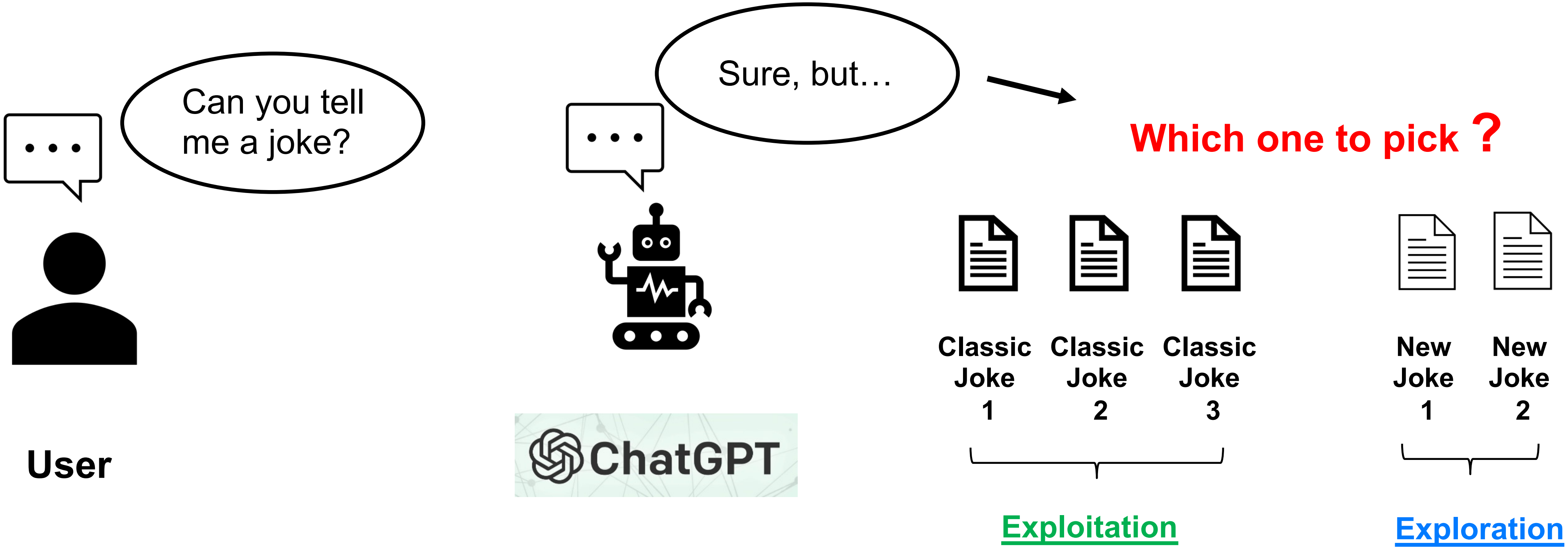
1. Slivkins, Aleksandrs. "Introduction to multi-armed bandits." *Foundations and Trends® in Machine Learning* 12.1-2 (2019): 1-286.

Exploitation VS Exploration in Sequential Recommendation

- Dilemma of **exploitation** and **exploration** is a **fundamental problem** in **sequential decision-making**.



Can you tell me a joke?

Sure, but…

Which one to pick ?

Classic Joke 1

Classic Joke 2

Classic Joke 3

New Joke 1

New Joke 2

Exploitation

Exploration

User

ChatGPT

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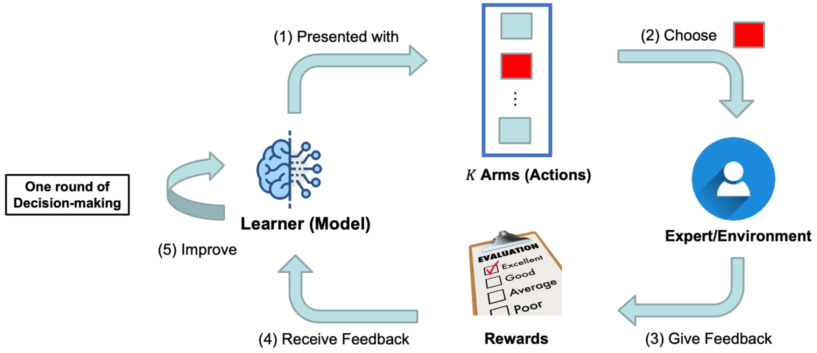
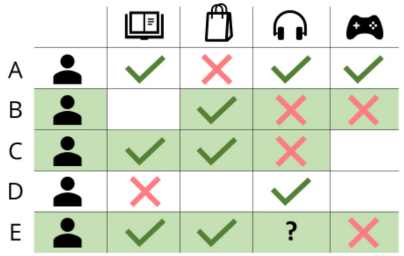
1. Slivkins, Aleksandrs. "Introduction to multi-armed bandits." *Foundations and Trends® in Machine Learning* 12.1-2 (2019): 1-286.

Advantages of Bandit-based Methods



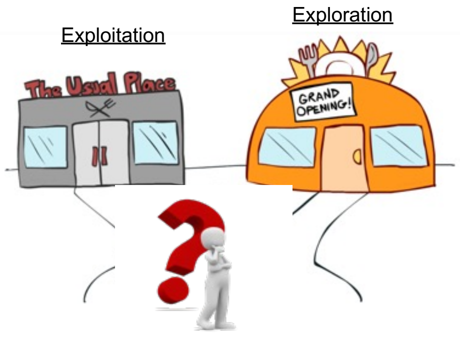
- ➢ **No Requirement for Large Collected Data.**



	📖	🛍	🎧	🎮
A	✓	✗	✓	✓
B		✓	✗	✗
C	✓	✓	✗	
D	✗		✓	
E	✓	✓	?	✗

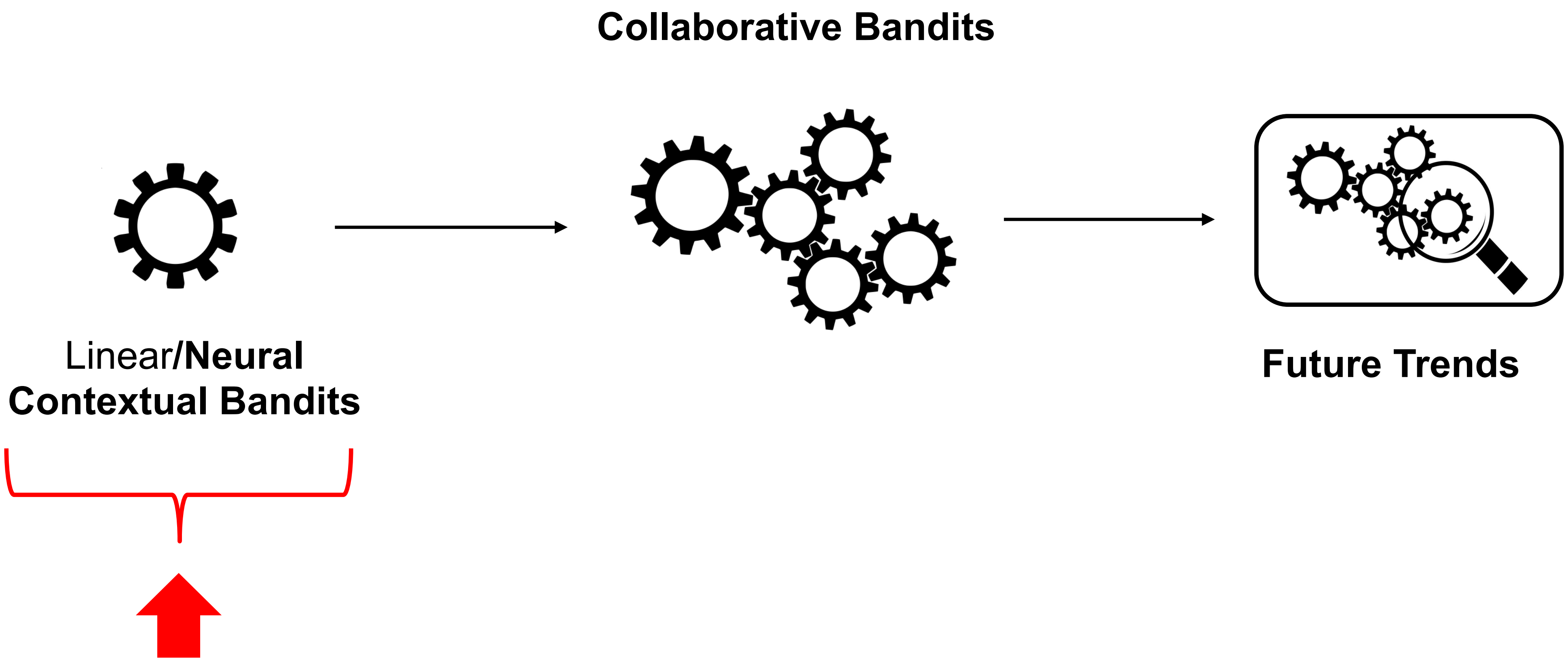
- ➢ **Adapt over Time.**

- ➢ **Explicit Exploration.**



1. Ban, Yikun, and Jingrui He. "Local clustering in contextual multi-armed bandits." WWW 2021.
2. Gao, Chongming, et al. "Alleviating matthew effect of offline reinforcement learning in interactive recommendation." ICLR 2023.

Tutorial Roadmap



Collaborative Bandits

Linear/**Neural**
Contextual Bandits

Future Trends

Neural Contextual Bandits: Roadmap

Fundamental Exploration

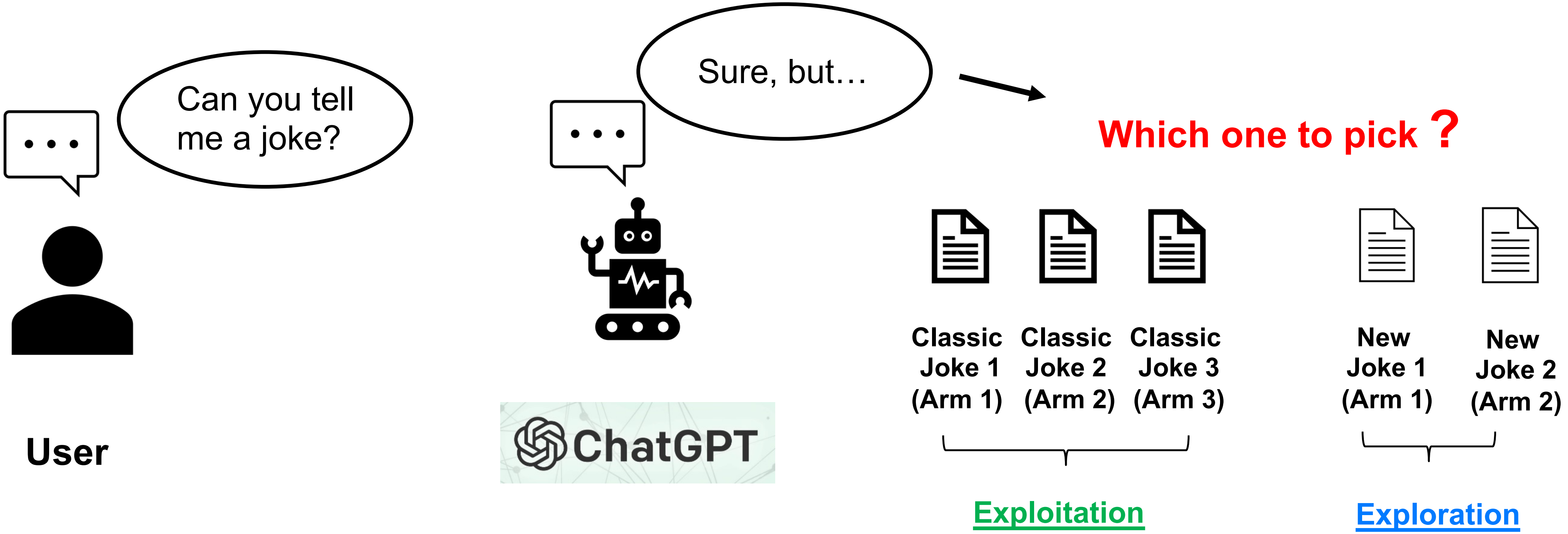
- Upper Confidence Bound
- Thompson Sampling
- Exploration Network

Efficient Exploration

- Neural Linear UCB
- Neural Network with Perturbed Reward
- Inverse Weight Gap Strategy

Background

- Popular existing exploration strategies.
  - ❑ **ϵ-greedy**: With probability $1 - \epsilon$, greedily choose one arm according to history; Otherwise, choose an arm randomly.



Can you tell me a joke?

User

Sure, but…

ChatGPT

Which one to pick ?

Classic Joke 1 (Arm 1)

Classic Joke 2 (Arm 2)

Classic Joke 3 (Arm 3)

New Joke 1 (Arm 1)

New Joke 2 (Arm 2)

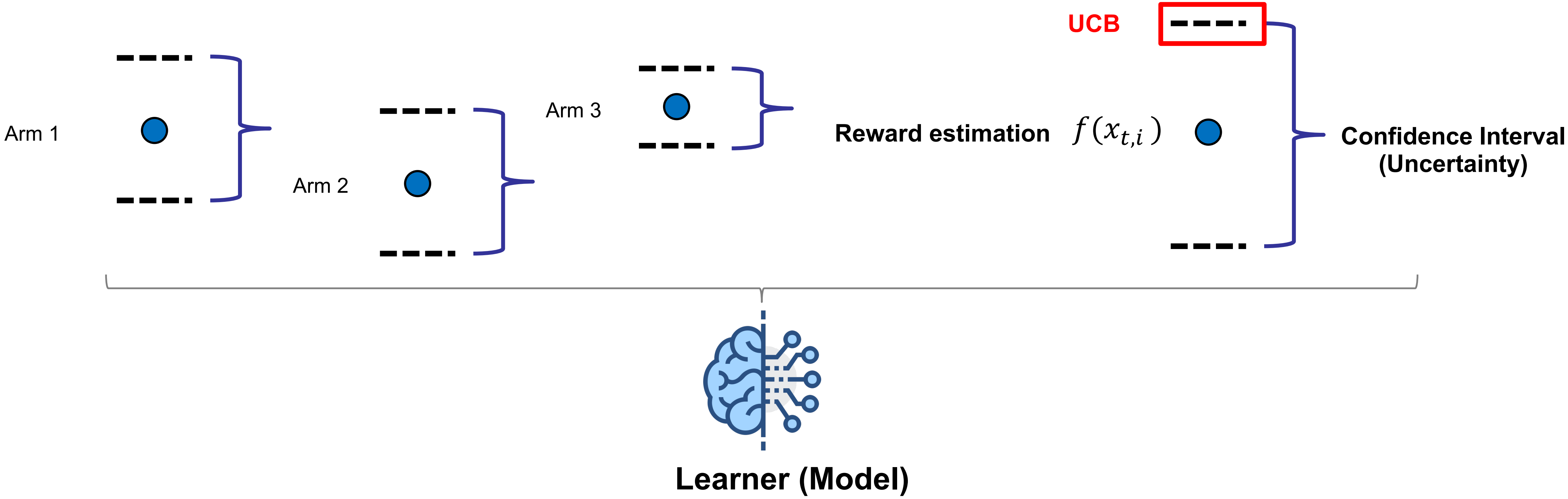
Exploitation

Exploration

1. Slivkins, Aleksandrs. "Introduction to multi-armed bandits." *Foundations and Trends® in Machine Learning* 12.1-2 (2019): 1-286.

Background

- Popular existing exploration strategies.
  - ☐ **ϵ-greedy**: With probability $1-\epsilon$, greedily choose one arm according to history; Otherwise, choose an arm randomly.
  - ☐ **Upper Confidence Bound [1]:**



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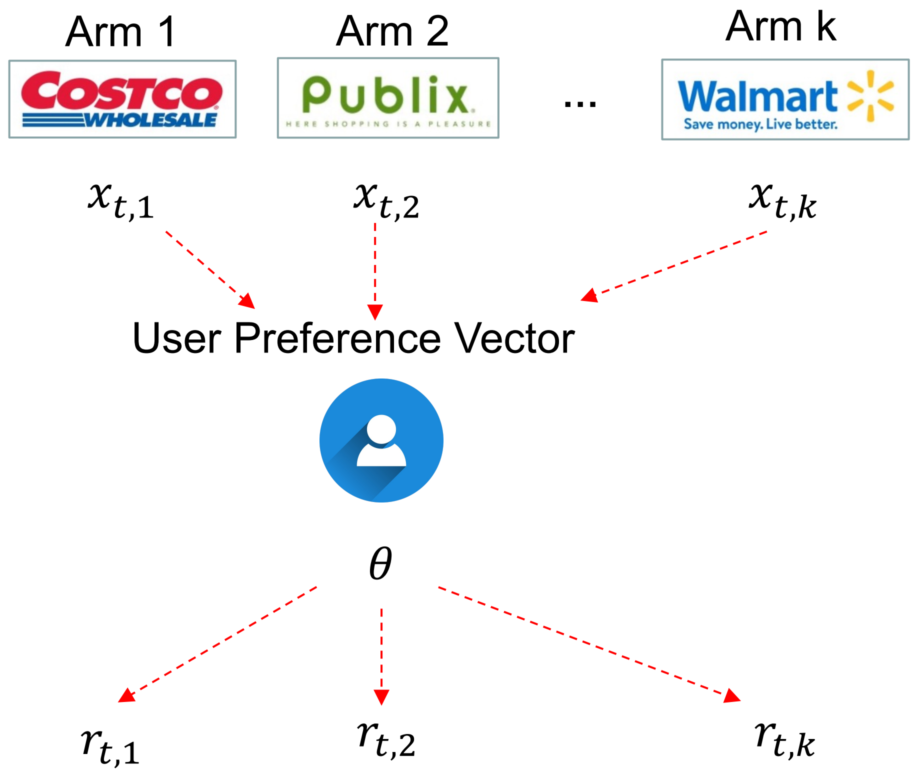
1. Slivkins, Aleksandrs. "Introduction to multi-armed bandits." *Foundations and Trends® in Machine Learning* 12.1-2 (2019): 1-286.

Linear Bandits: Joint Problem Definition

In round t: A user is serving



☐ **Arm Pool**: Arm 1, Arm 2, …, Arm k

$x_{t,1}$ $x_{t,2}$ $x_{t,k}$

d-dimensional Feature Vector

User Preference Vector

☐ **User Pool**:

d-dimensional Preference Vector (**Unknown**)

θ

☐ **Reward**:

$r_{t,1}$ $r_{t,2}$ $r_{t,k}$

Linear reward function

$$\boxed{r_{t,i} = \theta^T x_{t,i} + \eta_{t,i}}$$

Noise, ν-sub-Gaussian

---

1. Chu, Wei, et al. "Contextual bandits with linear payoff functions." AISTATS 2011.
2. Li, Lihong, Wei Chu, John Langford, and Robert E. Schapire. "A contextual-bandit approach to personalized news article recommendation." WWW 2010.

Linear Bandits: Disjoint Problem Definition

In round t: A user is serving

☐ **Arm Pool**: Arm 1, Arm 2, …, Arm k — d-dimensional Feature Vector

$x_{t,1}$ $x_{t,2}$ $x_{t,k}$

☐ **User Pool**: User 1, User 2, …, User n — d-dimensional Preference Vector (**Unknown**)

θ_1 θ_2 θ_n

☐ **Reward**: $r_{t,1}$ $r_{t,2}$ $r_{t,k}$ — Linear reward function

Given User j,

$$\boxed{r_{t,i} = \theta_j^T x_{t,i} + \eta_{t,i}}$$

Noise, ν-sub-Gaussian

---

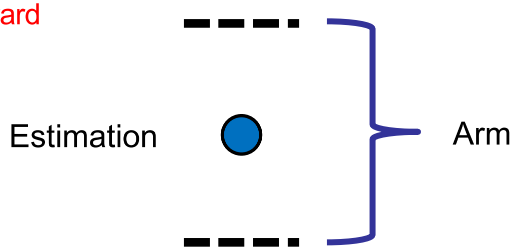
1. Chu, Wei, et al. "Contextual bandits with linear payoff functions." AISTATS 2011.
2. Li, Lihong, Wei Chu, John Langford, and Robert E. Schapire. "A contextual-bandit approach to personalized news article recommendation." WWW 2010.

Linear UCB: Algorithm

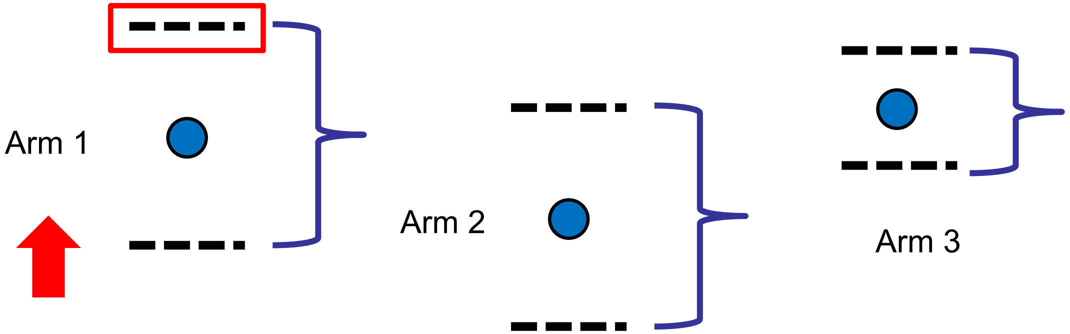
Confidence Interval:

$$\left|\underline{\hat{r}_{t,a}} - x_{t,a}^\top\theta^*\right| \le (\alpha+1)s_{t,a}.$$

Estimated Expected Reward

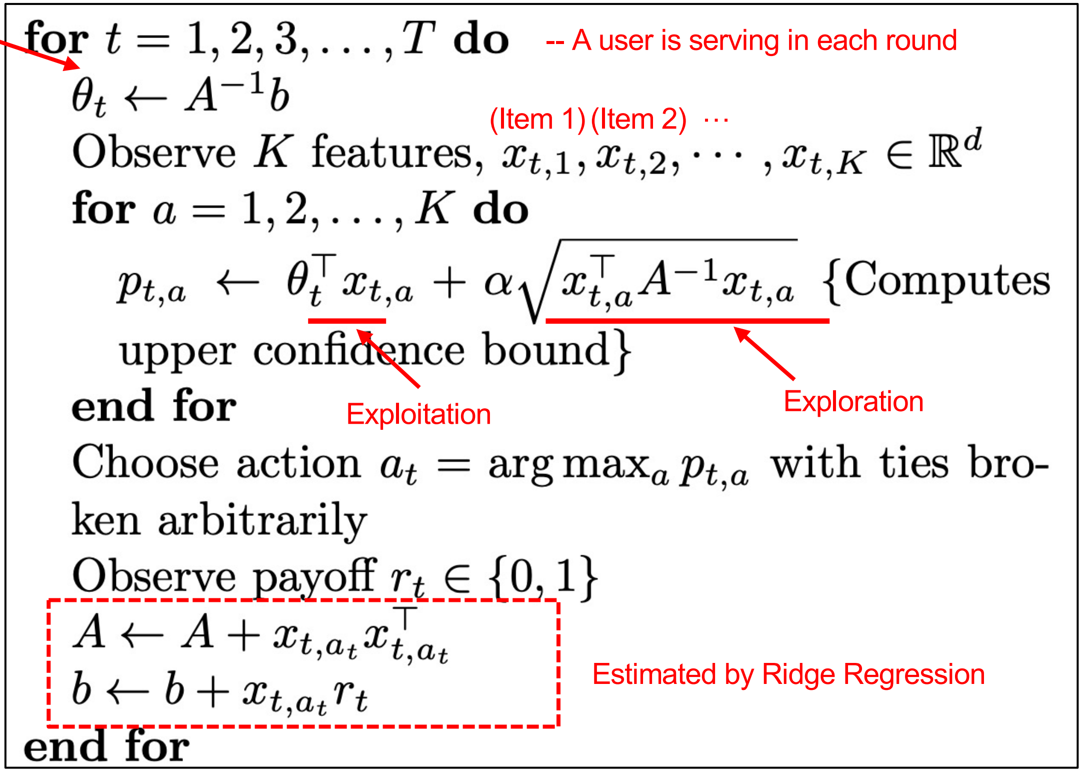


$$a_t \overset{\text{def}}{=} \arg\max_{a\in\mathcal{A}_t}\left(\mathbf{x}_{t,a}^\top\hat{\boldsymbol{\theta}}_a + \alpha\sqrt{\mathbf{x}_{t,a}^\top\mathbf{A}_a^{-1}\mathbf{x}_{t,a}}\right),$$



Joint Linear Models

Estimated by Ridge Regression



```
for $t = 1, 2, 3, \ldots, T$ do
    $\theta_t \leftarrow A^{-1}b$
    Observe $K$ features, $x_{t,1}, x_{t,2}, \cdots, x_{t,K} \in \mathbb{R}^d$
    for $a = 1, 2, \ldots, K$ do
        $p_{t,a} \leftarrow \underline{\theta_t^\top x_{t,a}} + \alpha\underline{\sqrt{x_{t,a}^\top A^{-1} x_{t,a}}}$ {Computes upper confidence bound}
    end for
    Choose action $a_t = \arg\max_a p_{t,a}$ with ties broken arbitrarily
    Observe payoff $r_t \in \{0, 1\}$
    $A \leftarrow A + x_{t,a_t} x_{t,a_t}^\top$
    $b \leftarrow b + x_{t,a_t} r_t$
end for
```

-- A user is serving in each round

(Item 1) (Item 2) ⋯

Exploitation

Exploration

Estimated by Ridge Regression

1. Chu, Wei, et al. "Contextual bandits with linear payoff functions." AISTATS 2011.
2. Li, Lihong, Wei Chu, John Langford, and Robert E. Schapire. "A contextual-bandit approach to personalized news article recommendation." WWW 2010.

Linear UCB: Regret Analysis

Confidence Interval:

Estimated by Ridge Regression

With high probability,

$$\left|\hat{r}_{t,a} - x_{t,a}^{\top}\theta^{*}\right| \leq (\alpha+1)s_{t,a}.$$

where

$$s_{t,a} \quad = \quad \sqrt{x_{t,a}^{\top} A_t^{-1} x_{t,a}} \in \mathbb{R}_{+}$$

$$\begin{aligned}
&\hat{r}_{t,a} - x_{t,a}^{\top}\theta^{*} \\
&\quad = \quad x_{t,a}^{\top}\theta_t - x_{t,a}^{\top}\theta^{*} \\
&\quad = \quad x_{t,a}^{\top}A_t^{-1}b_t - x_{t,a}^{\top}A_t^{-1}\left(I_d + D_t^{\top}D_t\right)\theta^{*} \\
&\quad = \quad x_{t,a}^{\top}A_t^{-1}D_t^{\top}y_t - x_{t,a}^{\top}A_t^{-1}\left(\theta^{*} + D_t^{\top}D_t\theta^{*}\right) \\
&\quad = \quad \underbrace{x_{t,a}^{\top}A_t^{-1}D_t^{\top}\left(y_t - D_t\theta^{*}\right)}_{\text{Bounded by Conf. Interval}} - \underbrace{x_{t,a}^{\top}A_t^{-1}\theta^{*}}_{\text{Bounded by Conf. Interval}},
\end{aligned}$$

Regret Upper Bound

$$O\left(\sqrt{Td\ln^{3}(KT\ln(T)/\delta)}\right).$$

The Number of Rounds

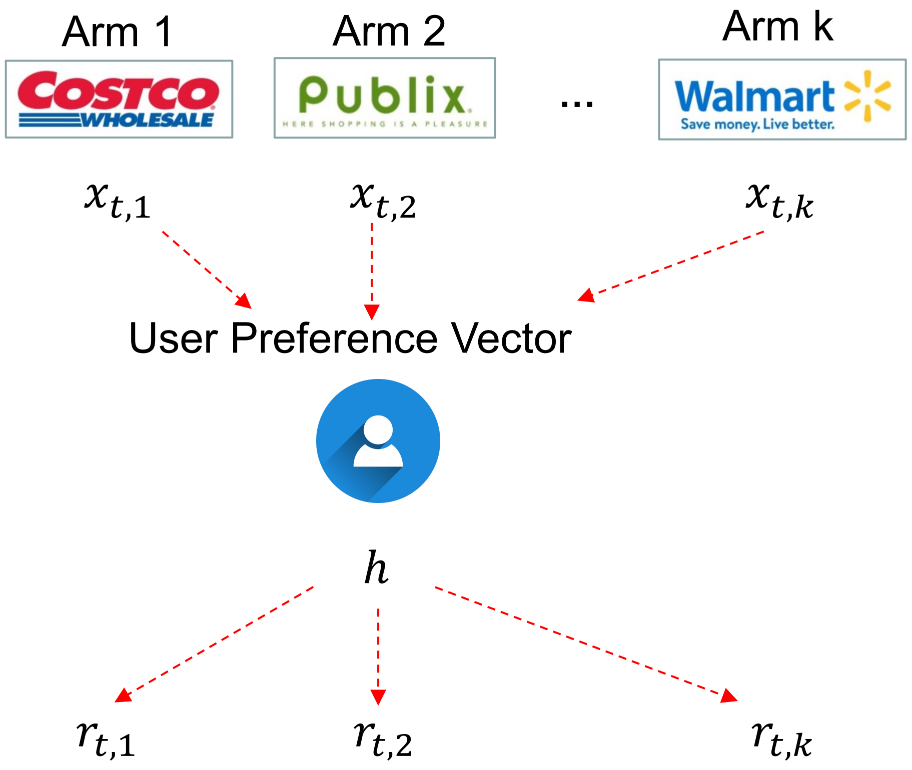
The Number of Items

Dimensionality of Item Context Vector

1. Chu, Wei, et al. "Contextual bandits with linear payoff functions." AISTATS 2011.
2. Li, Lihong, Wei Chu, John Langford, and Robert E. Schapire. "A contextual-bandit approach to personalized news article recommendation." WWW 2010.

Neural Bandits: Problem Formulation

In round t:



Arm 1 Arm 2 … Arm k

- **Arm Pool**: $x_{t,1}$ $x_{t,2}$ $x_{t,k}$

d-dimensional Feature Vector

User Preference Vector

- **User Pool**: h

Preference Function (**Unknown**)

- **Reward**: $r_{t,1}$ $r_{t,2}$ $r_{t,k}$

General reward function (Linear/Non-Linear)

$$\boxed{r_{t,i} = h(x_{t,i}) + \eta_{t,i}}$$

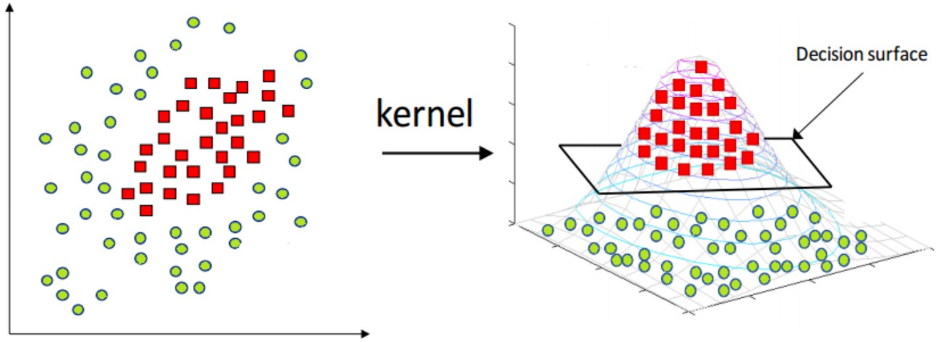
Sub-Gaussian Noise

---

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Neural Tangent Kernel

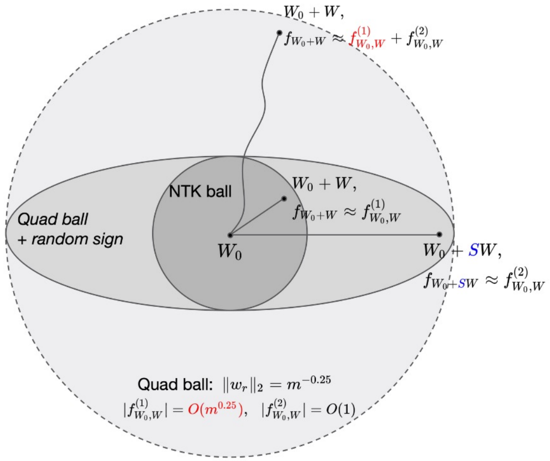
- A **sufficiently wide neural network** behaves like a **linearized model** governed by the derivative of network with respect to its parameters (**Gradient**).



https://www.geeksforgeeks.org/major-kernel-functions-in-support-vector-machine-svm/

Neural Tangent Kernel

$$\Theta(x, x'; \theta) = \nabla_\theta f(x; \theta) \cdot \nabla_\theta f(x'; \theta).$$

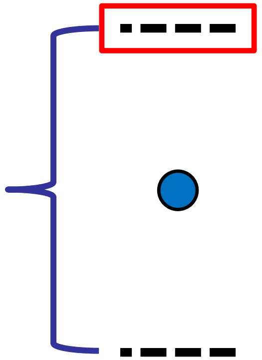


- With near-**infinite width**, Neural network behaves like a **kernel predictor** with Neural Tangent Kernel (NTK)

---

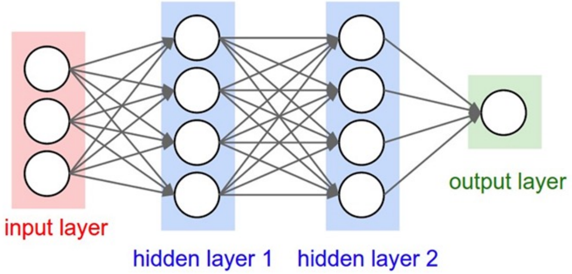
1. Jacot, Arthur, Franck Gabriel, and Clément Hongler. "Neural tangent kernel: Convergence and generalization in neural networks." NeurIPS 2018.
2. Allen-Zhu, Zeyuan, Yuanzhi Li, and Zhao Song. "A convergence theory for deep learning via over-parameterization." ICML 2019.

Neural UCB: Method



$U_{t,a}$

$$f(\mathbf{x};\boldsymbol{\theta}) = \sqrt{m}\mathbf{W}_L\sigma\Big(\mathbf{W}_{L-1}\sigma\big(\cdots\sigma(\mathbf{W}_1\mathbf{x})\big)\Big)$$



Gradient of f

$$U_{t,a} = \underbrace{f(\mathbf{x}_{t,a};\boldsymbol{\theta}_{t-1})}_{\text{mean}} + \gamma_{t-1}\underbrace{\sqrt{\boxed{\mathbf{g}(\mathbf{x}_{t,a};\boldsymbol{\theta}_{t-1})}^\top \mathbf{Z}_{t-1}^{-1}\mathbf{g}(\mathbf{x}_{t,a};\boldsymbol{\theta}_{t-1})/m}}_{\text{variance}}$$

Compared with LinUCB (Li et al. 2010)

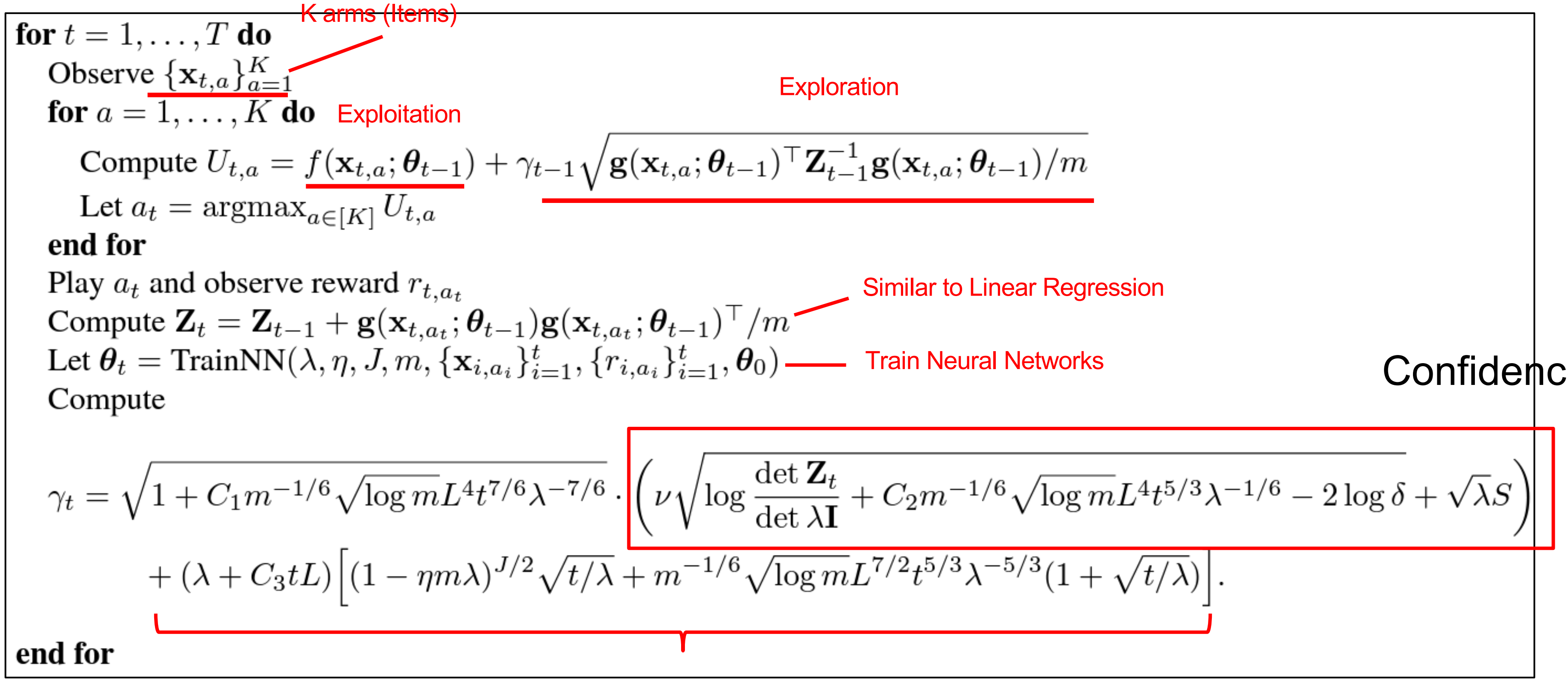
$$U_{t,a} = \underbrace{\langle \mathbf{x}_{t,a}, \boldsymbol{\theta}_{t-1}\rangle}_{\text{mean}} + \gamma_{t-1}\underbrace{\sqrt{\mathbf{x}_{t,a}^\top \mathbf{Z}_{t-1}^{-1}\mathbf{x}_{t,a}}}_{\text{variance}}$$

---

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Neural UCB: Workflow

- In each round, a user is serving



```
for $t = 1, \dots, T$ do
    Observe $\{\mathbf{x}_{t,a}\}_{a=1}^{K}$
    for $a = 1, \dots, K$ do
        Compute $U_{t,a} = \underbrace{f(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1})}_{\text{Exploitation}} + \underbrace{\gamma_{t-1}\sqrt{\mathbf{g}(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1})^\top \mathbf{Z}_{t-1}^{-1} \mathbf{g}(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1})/m}}_{\text{Exploration}}$
        Let $a_t = \text{argmax}_{a \in [K]} U_{t,a}$
    end for
    Play $a_t$ and observe reward $r_{t,a_t}$
    Compute $\mathbf{Z}_t = \mathbf{Z}_{t-1} + \mathbf{g}(\mathbf{x}_{t,a_t}; \boldsymbol{\theta}_{t-1}) \mathbf{g}(\mathbf{x}_{t,a_t}; \boldsymbol{\theta}_{t-1})^\top / m$
    Let $\boldsymbol{\theta}_t = \text{TrainNN}(\lambda, \eta, J, m, \{\mathbf{x}_{i,a_i}\}_{i=1}^{t}, \{r_{i,a_i}\}_{i=1}^{t}, \boldsymbol{\theta}_0)$
    Compute
        $\gamma_t = \sqrt{1 + C_1 m^{-1/6} \sqrt{\log m} L^4 t^{7/6} \lambda^{-7/6}} \cdot \boxed{\Big(\nu \sqrt{\log \frac{\det \mathbf{Z}_t}{\det \lambda \mathbf{I}} + C_2 m^{-1/6} \sqrt{\log m} L^4 t^{5/3} \lambda^{-1/6} - 2 \log \delta} + \sqrt{\lambda} S\Big)}$
            $+ \underbrace{(\lambda + C_3 t L)\Big[(1 - \eta m \lambda)^{J/2} \sqrt{t/\lambda} + m^{-1/6} \sqrt{\log m} L^{7/2} t^{5/3} \lambda^{-5/3} (1 + \sqrt{t/\lambda})\Big]}_{\text{Neural Function Approximation Error}}.$
end for
```

K arms (Items)

Exploration

Exploitation

Similar to Linear Regression

Train Neural Networks

Confidence Radius

Neural Function Approximation Error

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Neural UCB: Regret Analysis

- Definition of **NTK Matrix** on all observed contexts of T rounds.

$$\widetilde{\mathbf{H}}_{i,j}^{(1)} = \mathbf{\Sigma}_{i,j}^{(1)} = \langle \mathbf{x}^i, \mathbf{x}^j \rangle, \qquad \mathbf{A}_{i,j}^{(l)} = \begin{pmatrix} \mathbf{\Sigma}_{i,i}^{(l)} & \mathbf{\Sigma}_{i,j}^{(l)} \\ \mathbf{\Sigma}_{i,j}^{(l)} & \mathbf{\Sigma}_{j,j}^{(l)} \end{pmatrix},$$

$$\mathbf{\Sigma}_{i,j}^{(l+1)} = 2\mathbb{E}_{(u,v)\sim N(\mathbf{0},\mathbf{A}_{i,j}^{(l)})}\left[\sigma(u)\sigma(v)\right],$$

$$\widetilde{\mathbf{H}}_{i,j}^{(l+1)} = 2\widetilde{\mathbf{H}}_{i,j}^{(l)}\mathbb{E}_{(u,v)\sim N(\mathbf{0},\mathbf{A}_{i,j}^{(l)})}\left[\sigma'(u)\sigma'(v)\right] + \mathbf{\Sigma}_{i,j}^{(l+1)}.$$

Then, $\mathbf{H} = (\widetilde{\mathbf{H}}^{(L)} + \mathbf{\Sigma}^{(L)})/2$ is called the *neural tangent kernel (NTK)* matrix on the context set.

- Analyze dynamics of gradient and NTK regression.

Assumption: $\mathbf{H} \succeq \lambda_0 \mathbf{I}.$

- Satisfied if no two observed arm contexts are parallel.

Lemma: When neural network is wide enough,

Linear function w.r.t. Gradient

$$\underline{h(\mathbf{x}^i) = \langle \mathbf{g}(\mathbf{x}^i;\boldsymbol{\theta}_0), \boldsymbol{\theta}^* - \boldsymbol{\theta}_0 \rangle},$$
$$\sqrt{m}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}_0\|_2 \le \sqrt{2\mathbf{h}^\top \mathbf{H}^{-1}\mathbf{h}}, \qquad (5.1)$$

for all $i \in [TK]$.

---

1. Jacot, Arthur, Franck Gabriel, and Clément Hongler. "Neural tangent kernel: Convergence and generalization in neural networks." NeurIPS 2018.
2. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Neural UCB: Regret Analysis

Assumption: $\mathbf{H} \succeq \lambda_0 \mathbf{I}.$

Satisfied if *no* two contexts in $\{\mathbf{x}^i\}_{i=1}^{TK}$ are parallel.

$$h(\mathbf{x}^i) = \langle \mathbf{g}(\mathbf{x}^i; \boldsymbol{\theta}_0), \boldsymbol{\theta}^* - \boldsymbol{\theta}_0 \rangle,$$

$$\sqrt{m}\|\boldsymbol{\theta}^* - \boldsymbol{\theta}_0\|_2 \leq \sqrt{2\mathbf{h}^\top \mathbf{H}^{-1}\mathbf{h}},$$

$$S = \sqrt{2\mathbf{h}^\top \mathbf{H}^{-1}\mathbf{h}}$$

$$\widetilde{d} = \frac{\log\det(\mathbf{I} + \mathbf{H}/\lambda)}{\log(1 + TK/\lambda)}$$

<u>LinUCB</u>:

$$\widetilde{O}(d\sqrt{T})$$

Theorem

Let $\mathbf{h} = [h(\mathbf{x}^i)]_{i=1}^{TK} \in \mathbb{R}^{TK}$. *Set* $J = \widetilde{\boldsymbol{\Theta}}(TL/\lambda)$, $\eta = \Theta((mTL + m\lambda)^{-1})$ *and* $S = 2\sqrt{\mathbf{h}^\top \mathbf{H}^{-1}\mathbf{h}}$. *Under the overparameterized setting* ($m \gg 1$), *with probability at least* $1 - \delta$,

Upper Bound of Neural Parameters

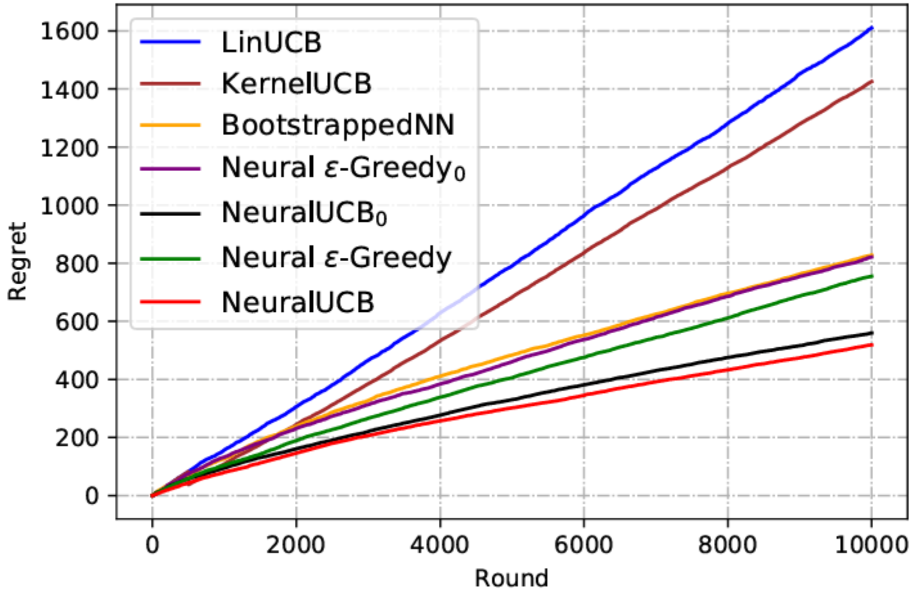
$$R_T = \widetilde{O}\left(\sqrt{\widetilde{d}T}\sqrt{\max\{\widetilde{d}, S^2\}}\right).$$

Effective dimension in NTK Space

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Neural UCB: Empirical Evaluation

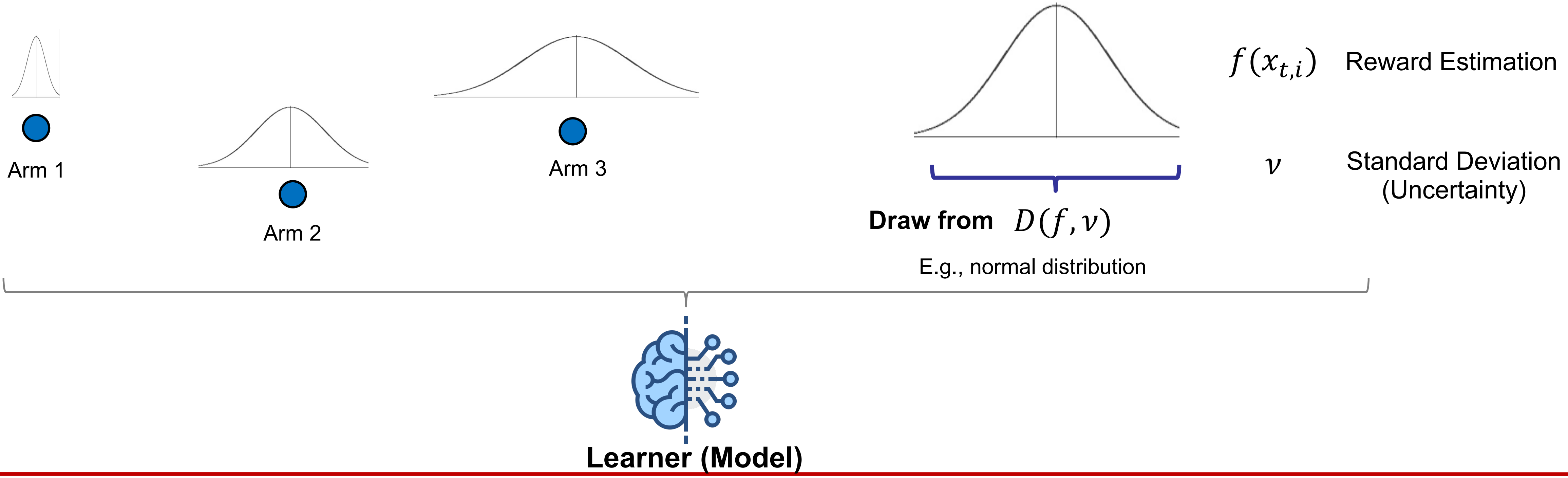
- NeuralUCB uses neural networks for exploitation, and gradient to explore.
- NeuralUCB achieve $\tilde{O}(\sqrt{T})$ regret upper bound, similar to LinearUCB.
- NeuralUCB generally outperforms linear contextual bandits.



1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.

Thompson Sampling

- Popular existing exploration strategies.
  - ☐ **ϵ-greedy**: With probability $1 - \epsilon$, greedily choose one arm according to history; Otherwise, choose an arm randomly.
  - ☐ **Upper Confidence Bound.**
  - ☐ **Thompson Sampling:**



Arm 1

Arm 2

Arm 3

$f(x_{t,i})$ Reward Estimation

ν Standard Deviation (Uncertainty)

Draw from $D(f, \nu)$

E.g., normal distribution

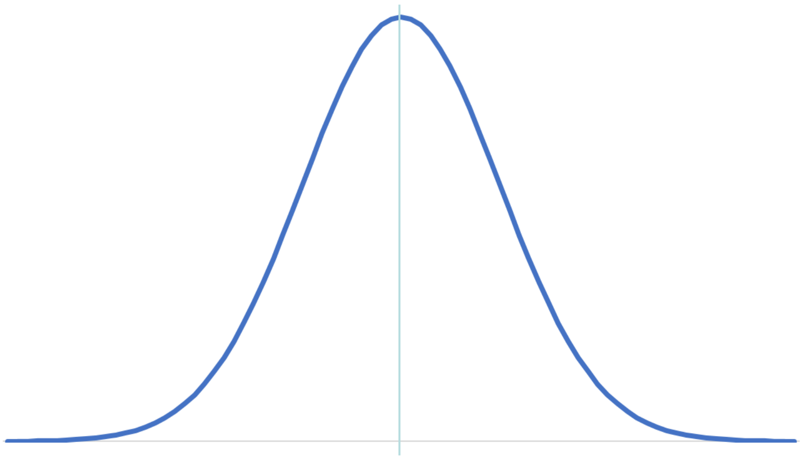
Learner (Model)

---

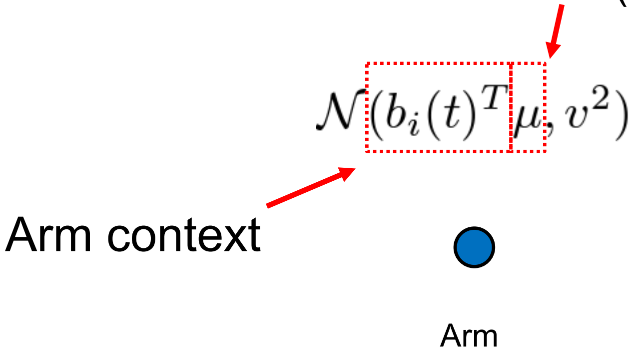
1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML, 2020.
2. Zhang, Weitong, et al. "Neural thompson sampling." ICLR 2021.

Linear Thompson Sampling

Reward Distribution (Gaussian Prior)



User Preference Parameter (Unknown)



$$\mathcal{N}(b_i(t)^T\mu, v^2)$$

Arm context

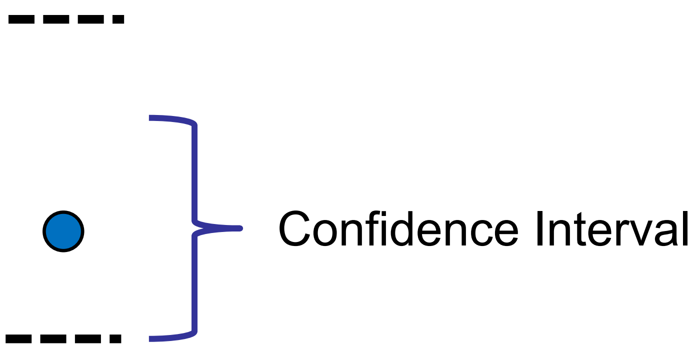
Arm

```
for all $t = 1, 2, \ldots,$ do
    Sample $\tilde{\mu}(t)$ from distribution $\mathcal{N}(\hat{\mu}, v^2 B^{-1})$.
    Play arm $a(t) := \arg\max_i b_i(t)^T \tilde{\mu}(t)$, and observe reward $r_t$.
    Update $B = B + b_{a(t)}(t) b_{a(t)}(t)^T$, $f = f + b_{a(t)}(t) r_t$, $\hat{\mu} = B^{-1} f$.
end for
```

Estimated User Preference

Sampled Reward

Estimated By Ridge Regression

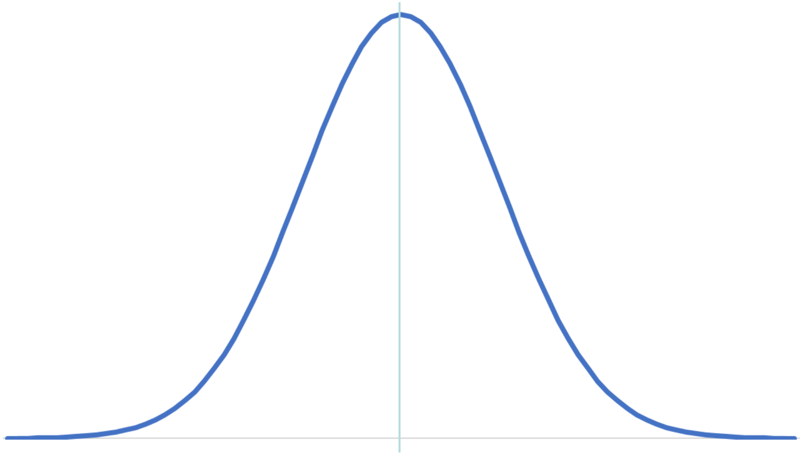


Confidence Interval

1. Agrawal, Shipra, and Navin Goyal. "Thompson sampling for contextual bandits with linear payoffs." ICML 2013.

Neural Thompson Sampling

Reward Distribution (Gaussian Prior)



$N(\, h\big(x_{t,k}\big)\, , v^2)$

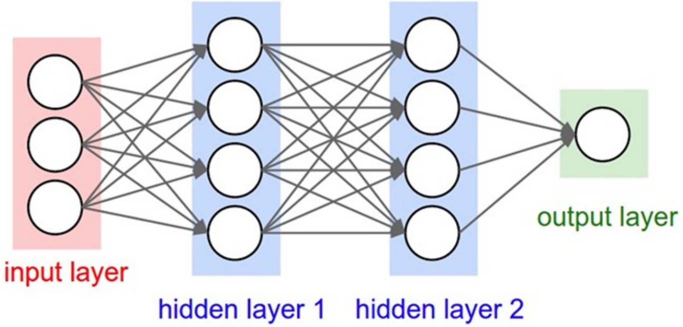
Expected Reward and Variance

Arm

Estimated Distribution:

$$\mathcal{N}(f(\mathbf{x}_{t,k}; \boldsymbol{\theta}_{t-1}), \nu^2 \sigma_{t,k}^2)$$

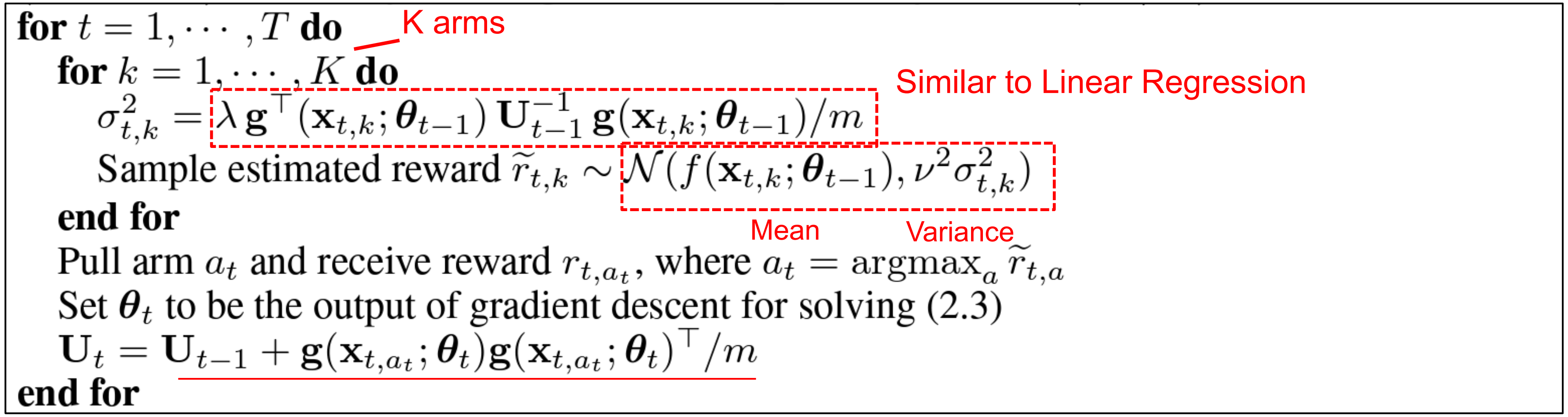
$$f(\mathbf{x}; \boldsymbol{\theta}) = \sqrt{m}\mathbf{W}_L \sigma\Big(\mathbf{W}_{L-1}\sigma\big(\cdots \sigma(\mathbf{W}_1\mathbf{x})\big)\Big)$$



1. Zhang, Weitong, Dongruo Zhou, Lihong Li, and Quanquan Gu. "Neural thompson sampling." ICLR 2021.

Neural Thompson Sampling

- In each round, a user is serving



```
for $t = 1, \cdots, T$ do
    for $k = 1, \cdots, K$ do
        $\sigma_{t,k}^2 = \boxed{\lambda \mathbf{g}^\top(\mathbf{x}_{t,k}; \boldsymbol{\theta}_{t-1}) \mathbf{U}_{t-1}^{-1} \mathbf{g}(\mathbf{x}_{t,k}; \boldsymbol{\theta}_{t-1})/m}$
        Sample estimated reward $\widetilde{r}_{t,k} \sim \boxed{\mathcal{N}(f(\mathbf{x}_{t,k}; \boldsymbol{\theta}_{t-1}), \nu^2 \sigma_{t,k}^2)}$
    end for
    Pull arm $a_t$ and receive reward $r_{t,a_t}$, where $a_t = \mathrm{argmax}_a \widetilde{r}_{t,a}$
    Set $\boldsymbol{\theta}_t$ to be the output of gradient descent for solving (2.3)
    $\mathbf{U}_t = \underline{\mathbf{U}_{t-1} + \mathbf{g}(\mathbf{x}_{t,a_t}; \boldsymbol{\theta}_t)\mathbf{g}(\mathbf{x}_{t,a_t}; \boldsymbol{\theta}_t)^\top/m}$
end for
```

K arms

Similar to Linear Regression

Mean

Variance

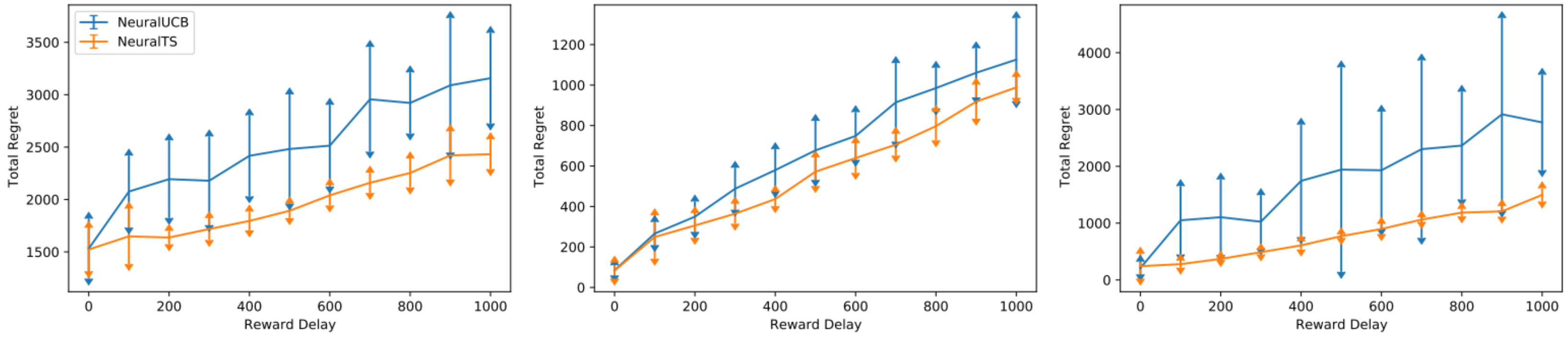
Compared to NeuralUCB:

$$U_{t,a} = f(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1}) + \gamma_{t-1} \boxed{\sqrt{\mathbf{g}(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1})^\top \mathbf{Z}_{t-1}^{-1} \mathbf{g}(\mathbf{x}_{t,a}; \boldsymbol{\theta}_{t-1})/m}}$$

---

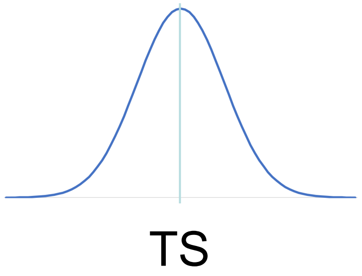
1. Zhang, Weitong, Dongruo Zhou, Lihong Li, and Quanquan Gu. "Neural thompson sampling." ICLR 2021.

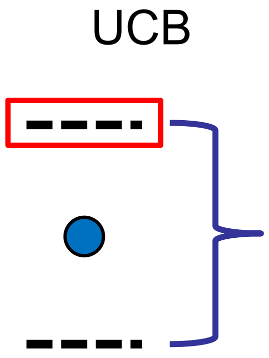
Neural Thompson Sampling



(a) MNIST (b) Mushroom (c) Shuttle

- ➢ NerualTS and NeuralUCB have **similar performance** when network is trained every iteration.
- ➢ NeuralTS is more robust than NeuralUCB when network is trained **in batch**.
- ➢ NeuralTS introduces more **robustness** in exploration.





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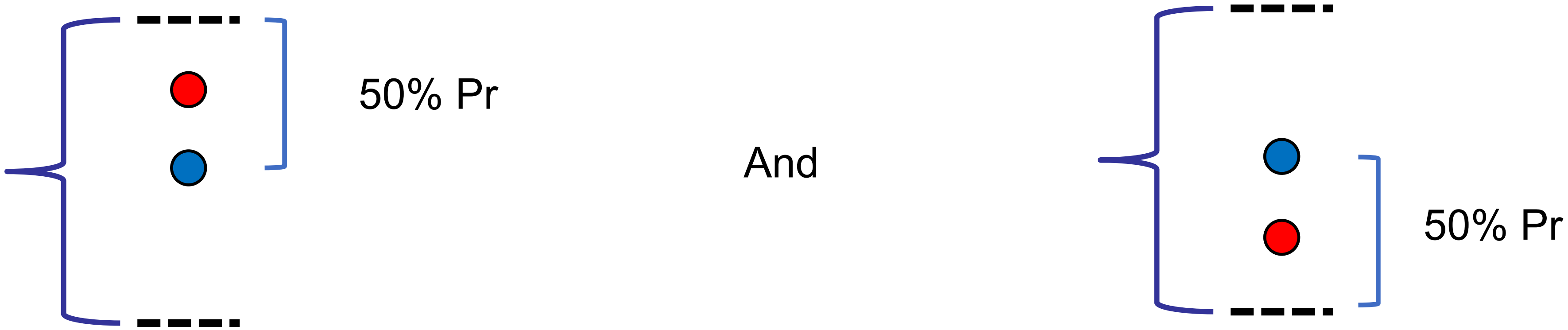
1. Zhang, Weitong, Dongruo Zhou, Lihong Li, and Quanquan Gu. "Neural thompson sampling." ICLR 2021.

EE-Net: Background

- UCB-based and TS-based exploration highly rely on large-deviation-based **statistical confidence interval**.
  - ☐ **Ideal** scenario:

● **Expected reward** ● **Estimated Reward**



Symmetric

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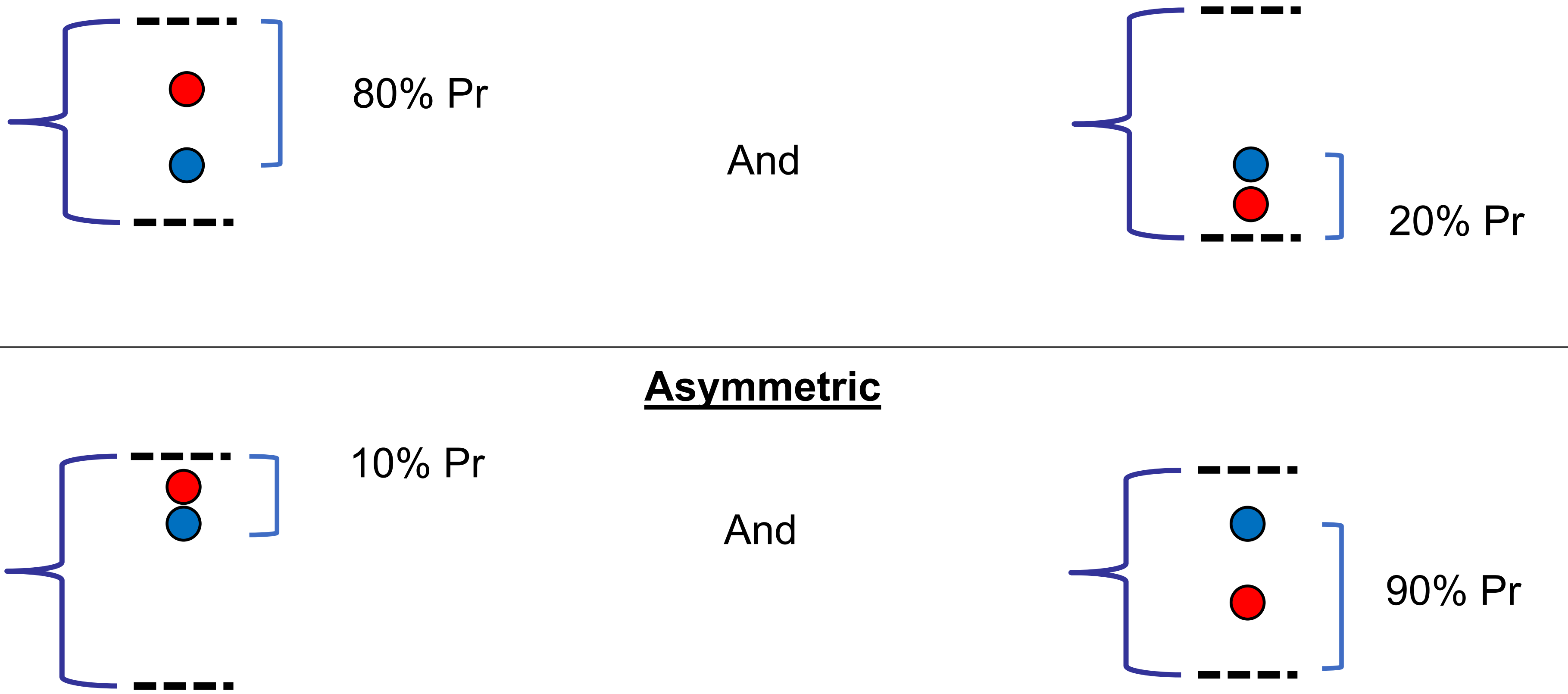
1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Motivation

- UCB-based and TS-based exploration highly rely on large-deviation-based **statistical confidence bound**.

☐ In practice, may be: ● Expected reward ● Estimated Reward



80% Pr

And

20% Pr

Asymmetric

10% Pr

And

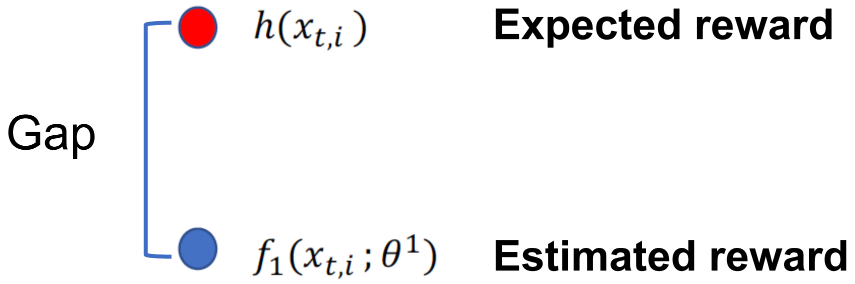
90% Pr

---

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Motivation

- **Why making exploration?**
  - Because we cannot make accurate prediction on a subset of data.
- **Goal of exploration**: Fill the **gap** between **expected reward** and **estimated reward**.



Gap

● $h(x_{t,i})$ **Expected reward**

● $f_1(x_{t,i}\,;\theta^1)$ **Estimated reward**

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1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Exploration Direction

- **Two types of exploration:** "**Upward**" exploration and "**downward**" Exploration.

● $h(x_{t,i})$ Expected reward

● $f_1(x_{t,i}\,;\theta^1)$ Estimation

Case 1: **Upward Exploration**

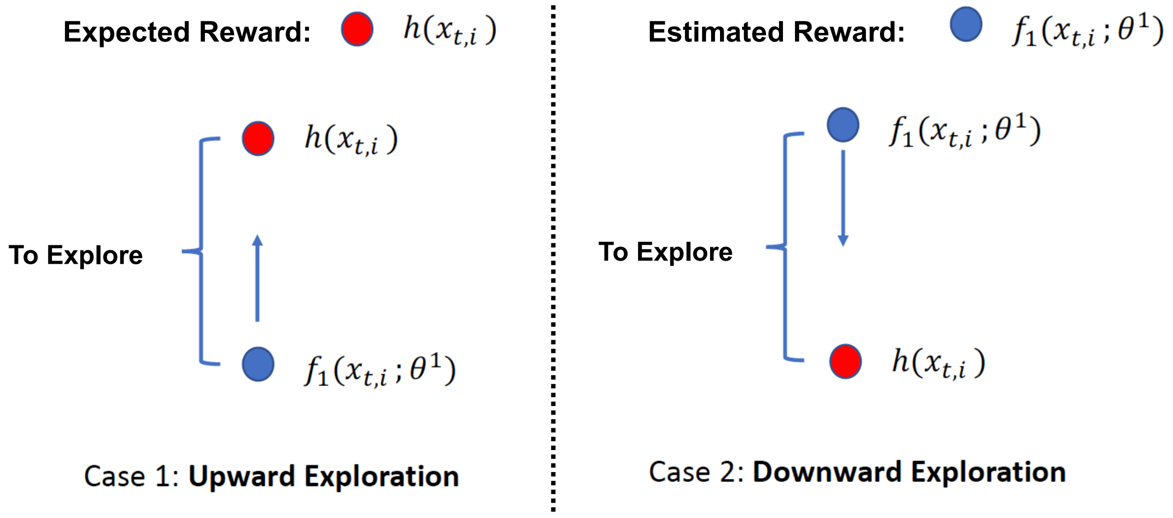
Case 2: **Downward Exploration**

Underestimation

Overestimation

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

Adapt to Exploration Direction is Challenging



Expected Reward: 🔴 $h(x_{t,i})$

Estimated Reward: 🔵 $f_1(x_{t,i}\,; \theta^1)$

Case 1: **Upward Exploration**

Case 2: **Downward Exploration**

Challenge:

Not 50% vs 50% !

Datasets	Upward Exploration	Downward Exploration
Mnist	**76.3%**	23.7%
Disin	29.1%	**70.9%**
MovieLens	**58.6%**	41.4%
Yelp	**55.3%**	44.7%

Pessimistic Model (Human)

Optimistic Model (Human)

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1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Solution

- Motivation: Can we have an **adaptive** exploration strategy for both “upward” and “downward” exploration?

- **Proposed solution**: We propose to use **another neural network to learn** the gap between expected reward and estimated reward (**potential gain**) **incorporating exploration direction**.

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Motivation and Solution

- Motivation: Can we have an **adaptive** exploration strategy for both “upward” and “downward” exploration?

- **Proposed solution**: We propose to use **another neural network to learn** the gap between expected reward and estimated reward (**potential gain**) **incorporating exploration direction**.

- **Exploitation neural network** f_1 to estimate reward:
  - Given an arm $x_{t,i}$,

$$f_1\left(x_{t,i};\theta^1\right) = \boldsymbol{W}_L\sigma(\boldsymbol{W}_{L-1}\sigma\left(\ldots\sigma(\boldsymbol{W}_1\cdot)\right))$$

  - $f_1(x_{t,i};\theta^1)$ is to **estimate expected reward** represented by some unknown function $h(x_{t,i})$.
  - In round t, θ^1 is **trained on data of past $t-1$ rounds**, using **gradient descent**.

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1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Exploration Neural Networks

- **Exploration neural network f_2** (novel component) to estimate potential gain:
  - Given an arm $x_{t,i}$ and its estimation $f_1(x_{t,i};\theta^1)$, **expected potential gain** is defined as:

$$h(x_{t,i}) - f_1(x_{t,i};\theta^1),$$

    where $h(x_{t,i})$ is the expected reward.

  - Thus, given the received reward $r_{t,i}$, **potential gain** is defined as:

$$r_{t,i} - f_1(x_{t,i};\theta^1),$$

    where $\mathbb{E}[r_{t,i}] = h(x_{t,i})$.

  - Potential gain has a good property: **Indicating exploration direction.**

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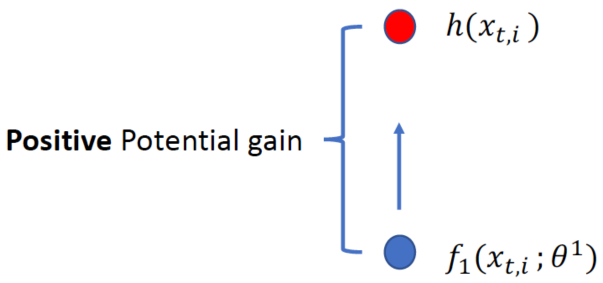
1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Exploration Neural Networks

- **Exploration neural network f_2** (novel component) to estimate potential gain:
  - Potential gain has good property: **indicating exploration direction.**

$$h(x_{t,i}) - f_1(x_{t,i}; \theta^1) > 0$$

$$h(x_{t,i}) - f_1(x_{t,i}; \theta^1) < 0$$

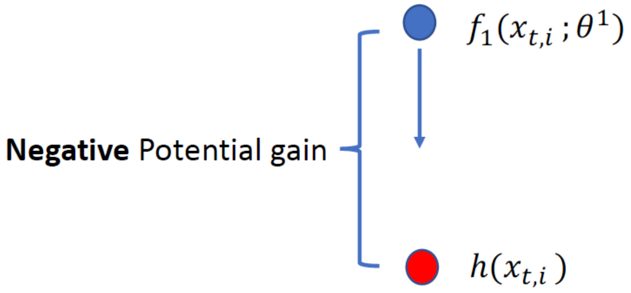


$h(x_{t,i})$

Positive Potential gain

$f_1(x_{t,i}; \theta^1)$

Case 1: **Upward Exploration**



$f_1(x_{t,i}; \theta^1)$

Negative Potential gain

$h(x_{t,i})$

Case 2: **Downward Exploration**

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1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Exploration Neural Networks

- **Exploration neural network f_2**(novel component) to estimate potential gain:
  - Label of f_2: $r_{t,i} - f_1(x_{t,i};)$

$$f_2\left(x_{t,i};\theta^2\right) = \boldsymbol{W}_L\sigma(\boldsymbol{W}_{L-1}\sigma\left(\ldots\sigma(\boldsymbol{W}_1\cdot)\right))$$

  - What is input of f_2?

---

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Exploration Neural Networks in Bandits

- **Exploration neural network f_2** (novel component) to estimate potential gain:
  - Input of f_2: **Gradient of f_1 with respect to θ^1**:

$$\nabla_{\theta^1} f(x_{t,i}; \theta^1)$$

- **Rational**:
  - ❑ Incorporate both **feature of input** and **discriminative information** of f_1.
  - ❑ Based on [2,3], f_1 has the following confidence bound:

$$|h(\mathbf{x}_{t,i}) - f_1(\mathbf{x}_{t,i}; \boldsymbol{\theta}^1_{t-1})| \leq \Psi(\nabla_{\boldsymbol{\theta}^1_{t-1}} f_1(\mathbf{x}_{t,i}; \boldsymbol{\theta}^1_{t-1})),$$

    Here, instead of choosing a fixed form Ψ, we use f_2 to learn it.

  - ❑ In this way, θ^2 is trained on $\{\nabla_{\theta^1} f(x_{t,i}; \theta^1_{\tau-1})\}_{\tau=1}^{t}$ to store historical information.

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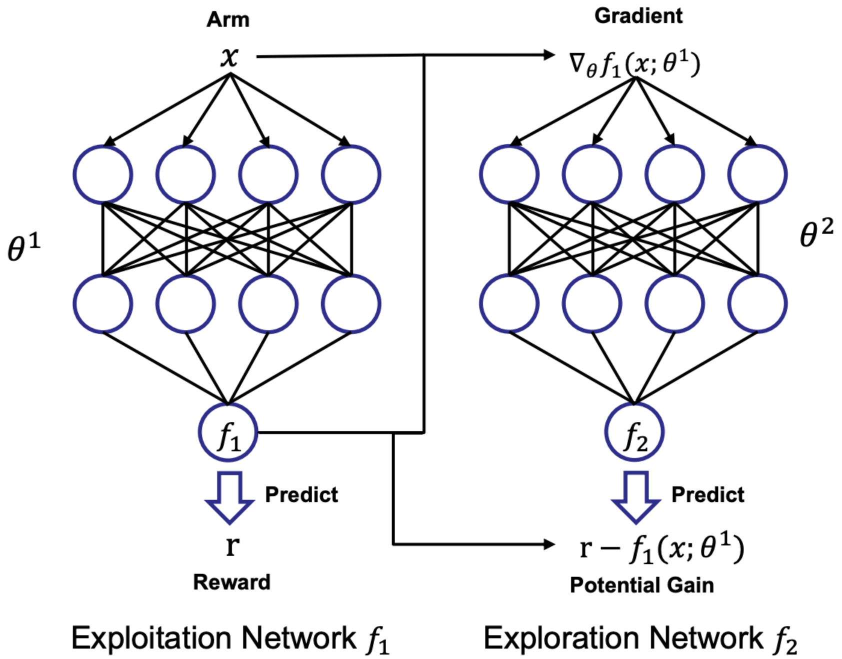
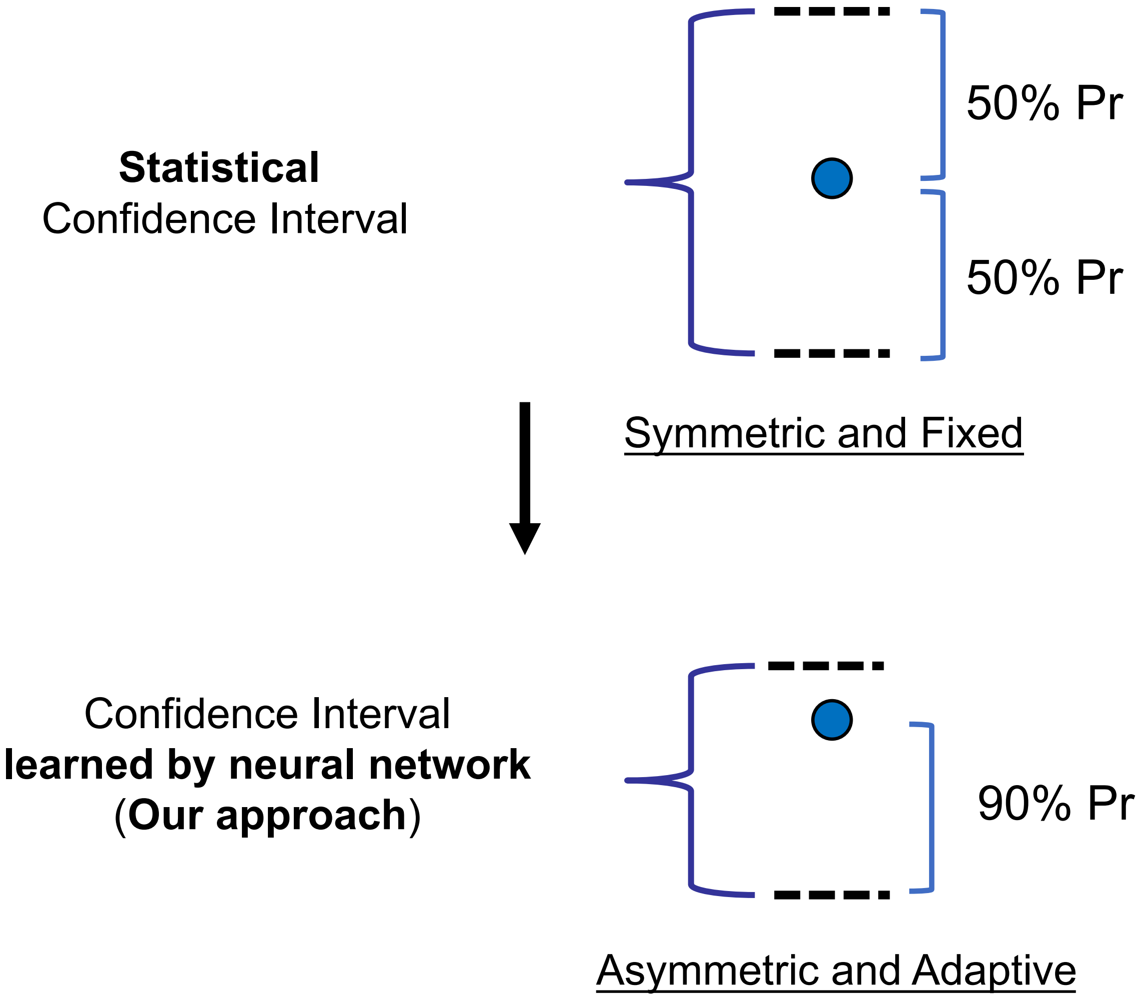
1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Overview



Methods	"Upward" Exploration	"Downward" Exploration
ϵ-Greedy	×	×
NeuralUCB	√	×
NeuralTS	Randomly	Randomly
EE-Net	√	√



Statistical
Confidence Interval

Symmetric and Fixed

Confidence Interval
learned by neural network
(**Our approach**)

Asymmetric and Adaptive

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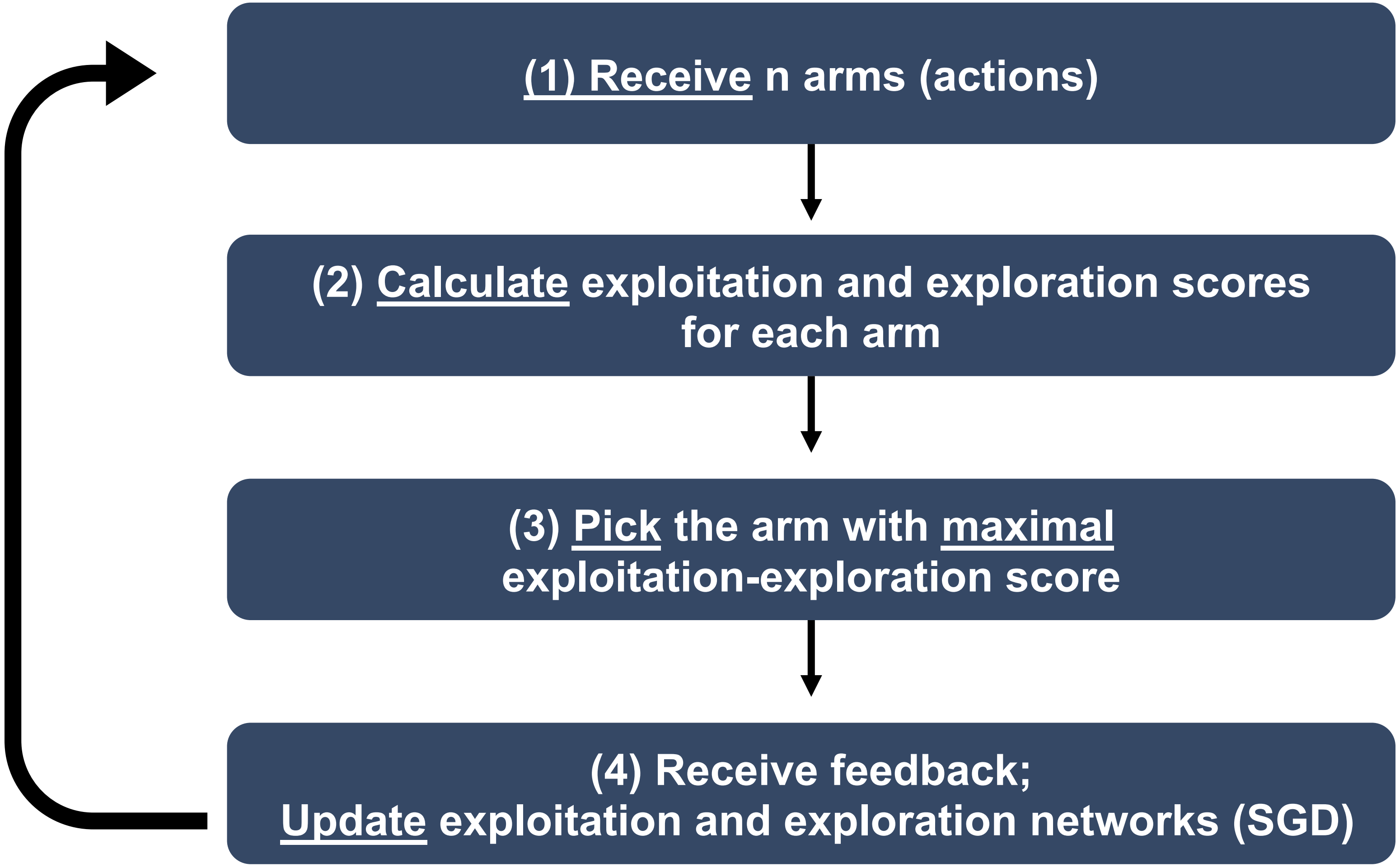
1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Workflow



(1) Receive n arms (actions)

(2) Calculate exploitation and exploration scores for each arm

(3) Pick the arm with maximal exploitation-exploration score

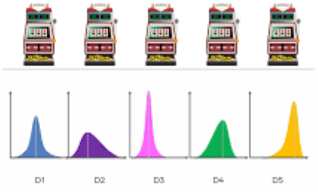
(4) Receive feedback;
Update exploitation and exploration networks (SGD)

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

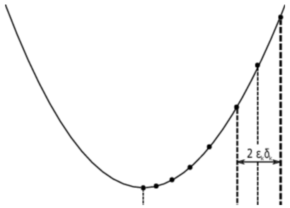
EE-Net: Theoretical Analysis

- Proof Workflow of NeuralUCB [1] and NeuralTS [2]:

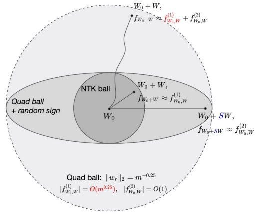
Gradient Descent in Bandits



Gradient Descent on Ridge Regression



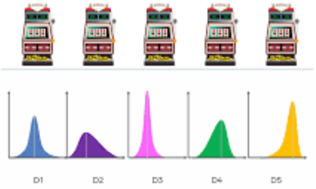
NTK Regression



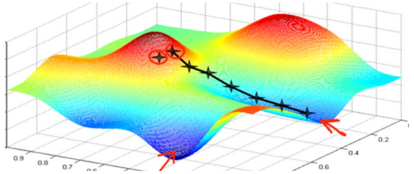
Expected Reward

- Proof Workflow of EE-Net [3,4]:

Gradient Descent in Bandits



Online Gradient Descent



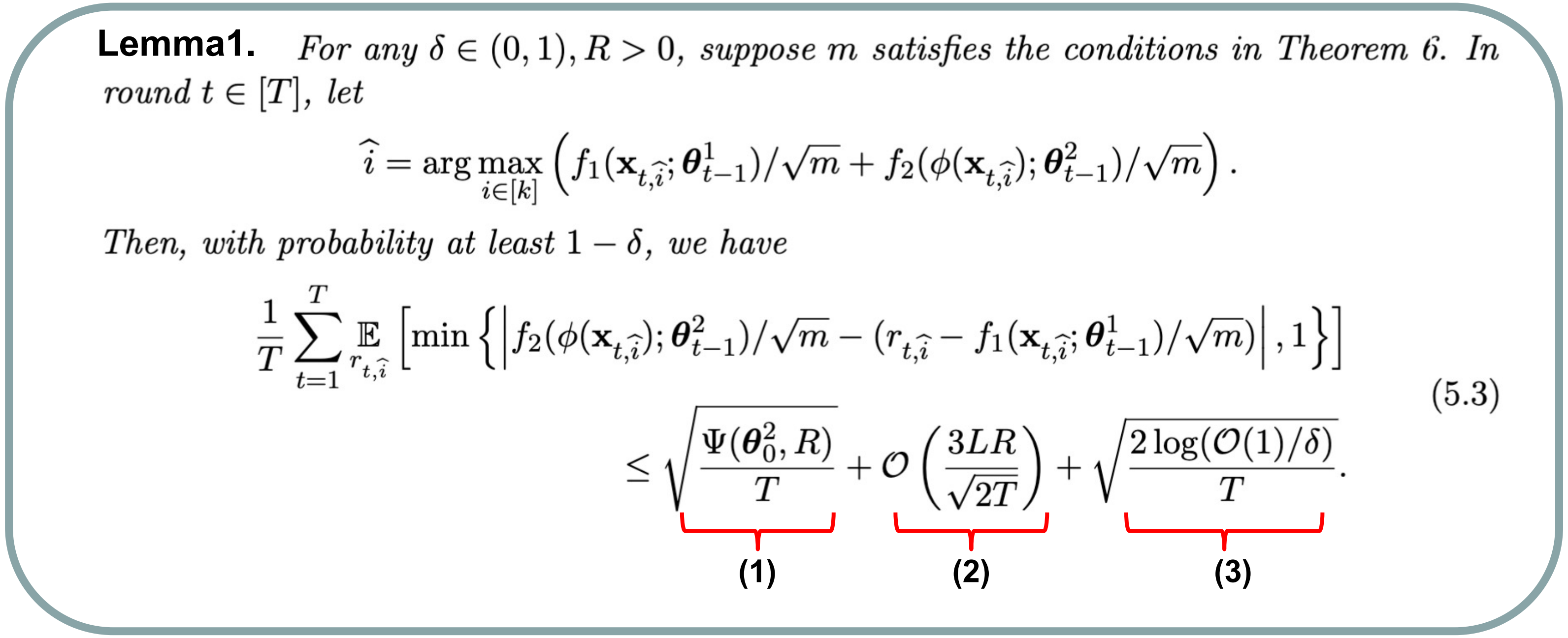
Expected Reward

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML, 2020.
2. Zhang, Weitong, et al. "Neural thompson sampling." ICLR 2021.
3. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
4. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Theoretical Analysis

Assumption 1: *For any* $t \in [T], i \in [n], \|\mathbf{x}_{t,i}\|_2 = 1,$ *and* $r_{t,i} \in [0, 1]$.

- Assumption 1 is standard and mild in analysis of over-parameterized neural networks.
- **No assumption** on distribution of arm contexts.
- Then, we have the following **average error bound for exploration network** f_2:



Lemma1. *For any* $\delta \in (0, 1), R > 0$, *suppose* m *satisfies the conditions in Theorem 6. In round* $t \in [T]$, *let*

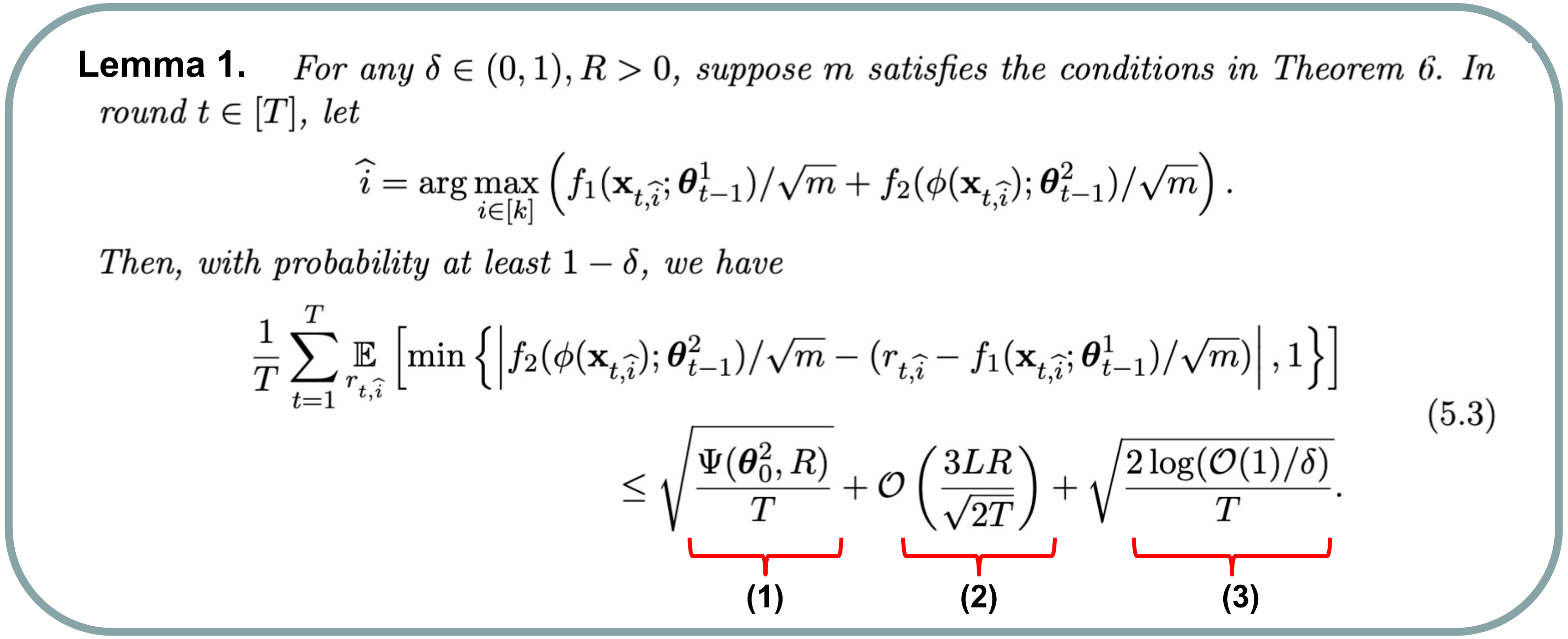
$$\hat{i} = \arg\max_{i \in [k]} \left(f_1(\mathbf{x}_{t,\hat{i}}; \boldsymbol{\theta}^1_{t-1})/\sqrt{m} + f_2(\phi(\mathbf{x}_{t,\hat{i}}); \boldsymbol{\theta}^2_{t-1})/\sqrt{m} \right).$$

Then, with probability at least $1 - \delta$, *we have*

$$\begin{aligned} &\frac{1}{T}\sum_{t=1}^{T} \mathop{\mathbb{E}}_{r_{t,\hat{i}}} \left[\min\left\{ \left| f_2(\phi(\mathbf{x}_{t,\hat{i}}); \boldsymbol{\theta}^2_{t-1})/\sqrt{m} - (r_{t,\hat{i}} - f_1(\mathbf{x}_{t,\hat{i}}; \boldsymbol{\theta}^1_{t-1})/\sqrt{m}) \right|, 1 \right\} \right] \\ &\leq \underbrace{\sqrt{\frac{\Psi(\boldsymbol{\theta}^2_0, R)}{T}}}_{\textbf{(1)}} + \underbrace{\mathcal{O}\left(\frac{3LR}{\sqrt{2T}}\right)}_{\textbf{(2)}} + \underbrace{\sqrt{\frac{2\log(\mathcal{O}(1)/\delta)}{T}}}_{\textbf{(3)}}. \end{aligned} \tag{5.3}$$

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Theoretical Analysis



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- **(1)** Complexity term Ψ: **Infimum of regression error** caused by function class $B(\theta^2, R)$:

$$\mathcal{B}(\boldsymbol{\theta}^2_0, R) = \{\widetilde{\boldsymbol{\theta}}^2 \in \mathbb{R}^p : \|\widetilde{\boldsymbol{\theta}}^2 - \boldsymbol{\theta}^2_0\|_2 \leq \mathcal{O}(\frac{R}{\sqrt{m}})\}. \qquad \Psi(\boldsymbol{\theta}^2_0, R) = \inf_{\widetilde{\boldsymbol{\theta}}^2 \in \mathcal{B}(\boldsymbol{\theta}^2_0, R)} \sum_{t=1}^{T} (f^2(\mathbf{x}_{t,\widehat{i}}; \widetilde{\boldsymbol{\theta}}^2) - r^2_{t,\widehat{i}})^2$$

- **(2) Price** of picking function class $B(\theta^2, R)$ controlled by radius R.
- **(3) Confidence bound** for predictions of f_2.

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

EE-Net: Regret Upper Bound

- Then, we have following regret upper bound $\tilde{O}(\sqrt{T})$ for EE-Net:

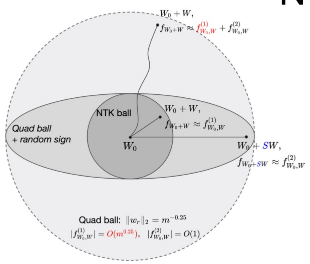
Theorem. *Let f_1, f_2 follow the setting of f (Eq. (5.1)) with the same width m and depth L. Suppose $m \geq \Omega\left(poly(T, L, R, \log(1/\delta))\right)$, $\eta_1 = \eta_2 = \frac{\sqrt{\nu}R}{m\sqrt{T}}$ and $\Psi(\boldsymbol{\theta}_0^2, R) \& \Psi^*(\boldsymbol{\theta}_0^2, R) \leq \Psi$. Then, for any $\delta \in (0,1), R > 0$, with probability at least $1-\delta$ over the initialization, there exists a constant ν, such that the pseudo regret of Algorithm 1 in T rounds satisfies*

$$\mathbf{R}_T \leq \sqrt{T} \cdot \mathcal{O}\left(RL + \sqrt{\Psi} + 2\sqrt{2\log(\mathcal{O}(1)/\delta)}\right) + \mathcal{O}(1) \qquad (5.2)$$

- Compared to existing works NeuralUCB [3] and NeuralTS [4]:

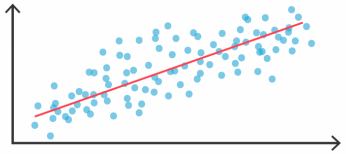
$$\mathbf{R}_T \leq \mathcal{O}\left(\sqrt{\tilde{d}T\log T + S^2}\right) \cdot \mathcal{O}\left(\sqrt{\tilde{d}\log T}\right), \quad \text{and} \quad \tilde{d} = \frac{\log\det(\mathbf{I} + \mathbf{H}/\lambda)}{\log(1 + Tn/\lambda)}$$

NTK

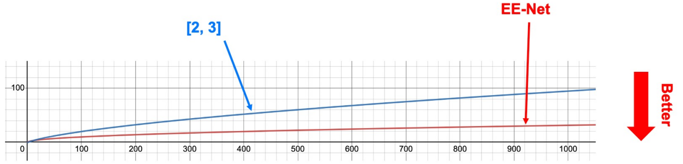


vs

Regression



1) **[Better Interpretability]:** Have the similar complexity term but Ψ **easier to interpret**.
2) **[Contexts]:** Allow arm contexts to be **repeatedly observed**.
3) **[Tighter Bound]:** EE-Net **improves by a multiplicative factor** $\log T$.



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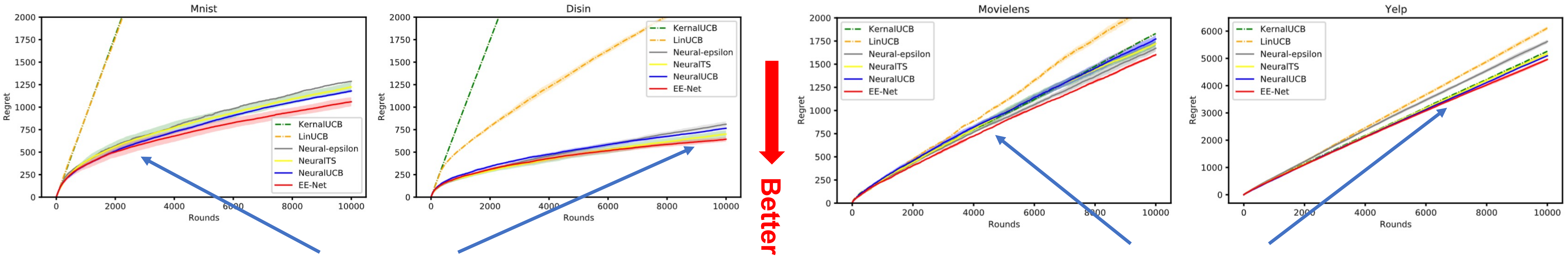
1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.
3. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML, 2020.
4. Zhang, Weitong, et al. "Neural thompson sampling." ICLR 2021.

EE-Net: Empirical Experiments



EE-Net

EE-Net

- Setup:
  - ❑ Classification and recommendation dataset.
  - ❑ 5 state-of-the-art baselines including ϵ-greedy, UCB, TS exploration strategy.
  - ❑ All methods have the same exploitation network f_1.

- **EE-Net** achieves **substantial improvements,** because **all improvements purely come from exploration!**

---

1. Ban, Yikun, et al. "EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits." ICLR 2022.
2. Ban, Yikun, et al. "Neural Exploitation and Exploration of Contextual Bandits." JMLR 2024.

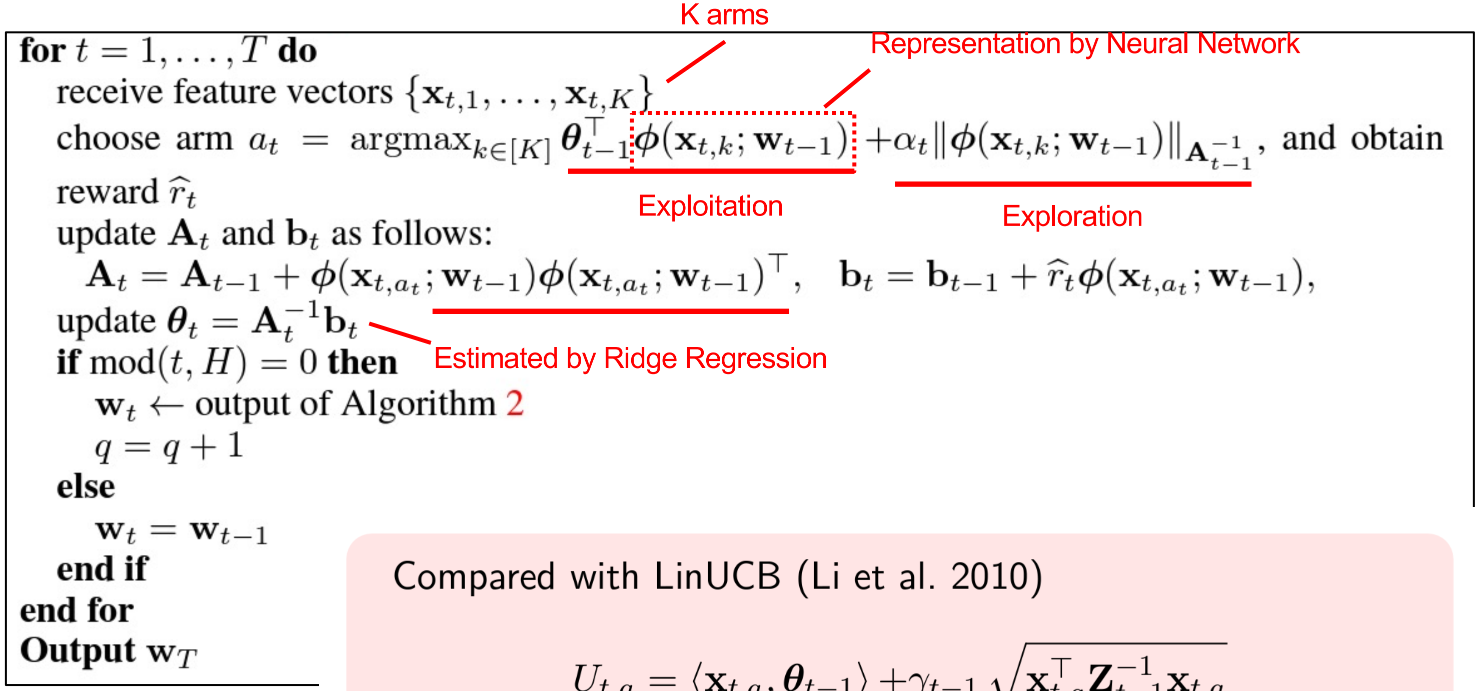
Neural Contextual Bandits: Roadmap

Fundamental Exploration
- Upper Confidence Bound
- Thompson Sampling
- Exploration Network

Efficient Exploration
- Neural Linear UCB
- Neural Network with Perturbed Reward
- Inverse Weight Gap Strategy

Neural Linear UCB

- In each round, a user is serving



```
for $t = 1, \dots, T$ do
    receive feature vectors $\{\mathbf{x}_{t,1}, \dots, \mathbf{x}_{t,K}\}$
    choose arm $a_t = \mathrm{argmax}_{k \in [K]} \boldsymbol{\theta}_{t-1}^\top \phi(\mathbf{x}_{t,k}; \mathbf{w}_{t-1}) + \alpha_t \|\phi(\mathbf{x}_{t,k}; \mathbf{w}_{t-1})\|_{\mathbf{A}_{t-1}^{-1}}$, and obtain reward $\widehat{r}_t$
    update $\mathbf{A}_t$ and $\mathbf{b}_t$ as follows:
      $\mathbf{A}_t = \mathbf{A}_{t-1} + \phi(\mathbf{x}_{t,a_t}; \mathbf{w}_{t-1})\phi(\mathbf{x}_{t,a_t}; \mathbf{w}_{t-1})^\top, \quad \mathbf{b}_t = \mathbf{b}_{t-1} + \widehat{r}_t \phi(\mathbf{x}_{t,a_t}; \mathbf{w}_{t-1}),$
    update $\boldsymbol{\theta}_t = \mathbf{A}_t^{-1}\mathbf{b}_t$
    if mod$(t, H) = 0$ then
        $\mathbf{w}_t \leftarrow$ output of Algorithm 2
        $q = q + 1$
    else
        $\mathbf{w}_t = \mathbf{w}_{t-1}$
    end if
end for
Output $\mathbf{w}_T$
```

Compared with LinUCB (Li et al. 2010)

$$U_{t,a} = \underbrace{\langle \mathbf{x}_{t,a}, \boldsymbol{\theta}_{t-1} \rangle}_{\text{mean}} + \gamma_{t-1} \underbrace{\sqrt{\mathbf{x}_{t,a}^\top \mathbf{Z}_{t-1}^{-1} \mathbf{x}_{t,a}}}_{\text{variance}}$$

- Update Neural Network Parameter:

  Loss function:

$$\mathcal{L}_q(\mathbf{w}) = \sum_{i=1}^{qH} \left(\boldsymbol{\theta}_i^\top \phi(\mathbf{x}_{i,a_i}; \mathbf{w}) - \widehat{r}_i\right)^2.$$

- Gradient Descent.

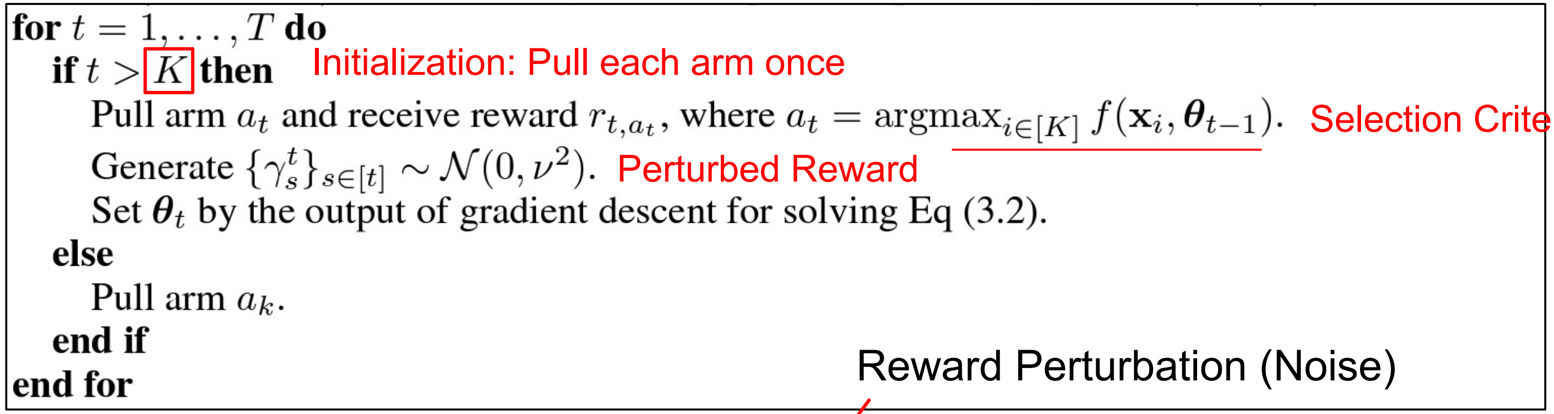
$$\phi(\mathbf{x}; \mathbf{w}) = \sqrt{m}\sigma(\mathbf{W}_L \sigma(\mathbf{W}_{L-1} \cdots \sigma(\mathbf{W}_1 \mathbf{x}) \cdots)).$$

---

1. Xu, Pan, et al. “Neural contextual bandits with deep representation and shallow exploration.” ICLR 2022.

Neural Bandit With Perturbed Reward

- In each round, a user is serving



```
for $t = 1, \ldots, T$ do
    if $t > \boxed{K}$ then
        Pull arm $a_t$ and receive reward $r_{t,a_t}$, where $a_t = \underline{\mathrm{argmax}_{i\in[K]} f(\mathbf{x}_i, \boldsymbol{\theta}_{t-1})}$.
        Generate $\{\gamma_s^t\}_{s\in[t]} \sim \mathcal{N}(0, \nu^2)$.
        Set $\boldsymbol{\theta}_t$ by the output of gradient descent for solving Eq (3.2).
    else
        Pull arm $a_k$.
    end if
end for
```

Initialization: Pull each arm once

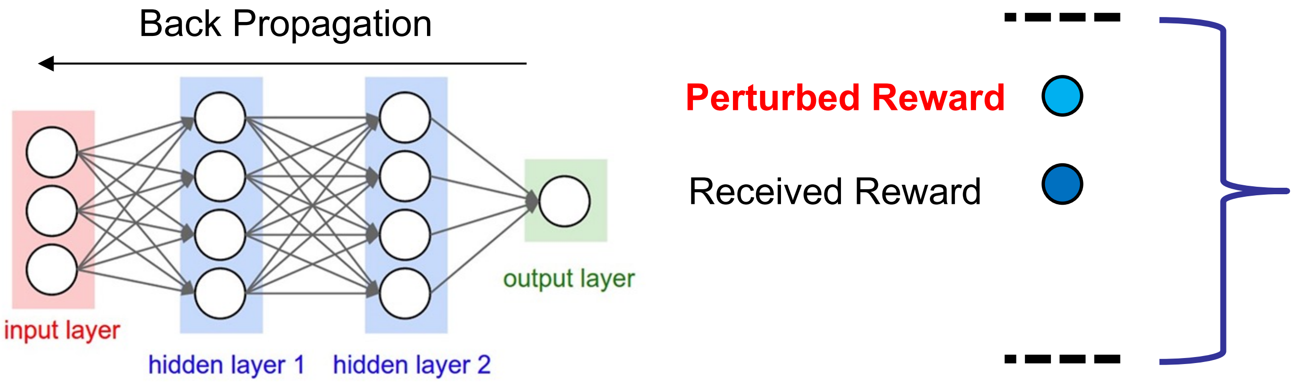
Selection Criterion

Perturbed Reward

Reward Perturbation (Noise)

$$\min_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}) = \sum_{s=1}^{t} \big(f(\mathbf{x}_{a_s};\boldsymbol{\theta}) - (r_{s,a_s} + \boxed{\gamma_s^t})\big)^2/2 + m\lambda\|\boldsymbol{\theta} - \boldsymbol{\theta}_0\|_2^2/2$$

Implicit Exploration:



1. Jia, Yiling, Weitong Zhang, Dongruo Zhou, Quanquan Gu, and Hongning Wang. "Learning neural contextual bandits through perturbed rewards." ICLR 2022.

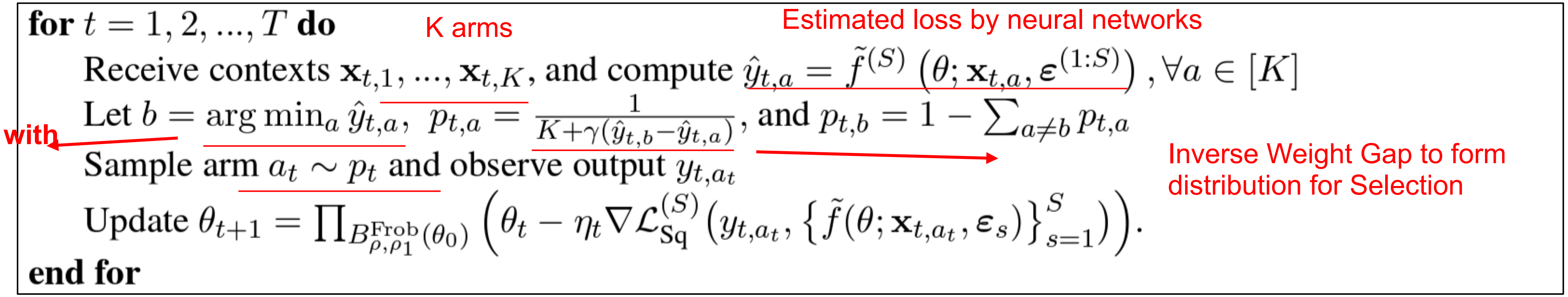
Neural SquareCB: Inverse Gap Strategy

- In each round, a user is serving

Special Case: $y = 1 - r$

Estimated loss by neural networks

K arms



```
for $t = 1, 2, ..., T$ do
    Receive contexts $\mathbf{x}_{t,1}, ..., \mathbf{x}_{t,K}$, and compute $\hat{y}_{t,a} = \tilde{f}^{(S)}\left(\theta; \mathbf{x}_{t,a}, \boldsymbol{\varepsilon}^{(1:S)}\right)$ , $\forall a \in [K]$
    Let $b = \arg\min_a \hat{y}_{t,a}$, $p_{t,a} = \frac{1}{K+\gamma(\hat{y}_{t,b}-\hat{y}_{t,a})}$, and $p_{t,b} = 1 - \sum_{a \neq b} p_{t,a}$
    Sample arm $a_t \sim p_t$ and observe output $y_{t,a_t}$
    Update $\theta_{t+1} = \prod_{B^{\mathrm{Frob}}_{\rho,\rho_1}(\theta_0)} \left(\theta_t - \eta_t \nabla \mathcal{L}^{(S)}_{\mathrm{Sq}}\left(y_{t,a_t}, \left\{\tilde{f}(\theta; \mathbf{x}_{t,a_t}, \boldsymbol{\varepsilon}_s)\right\}_{s=1}^{S}\right)\right)$.
end for
```

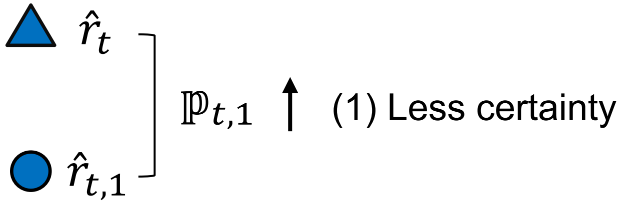
Equal to the **arm with maximal reward**

Inverse Weight Gap to form distribution for Selection

$$\hat{r}_t = \arg\max f(x_{t,i}; \theta_t)$$

$$\mathbb{p}_{t,i} \propto \frac{1}{\hat{r}_t - \hat{r}_{t,i}}$$

Selection Probability

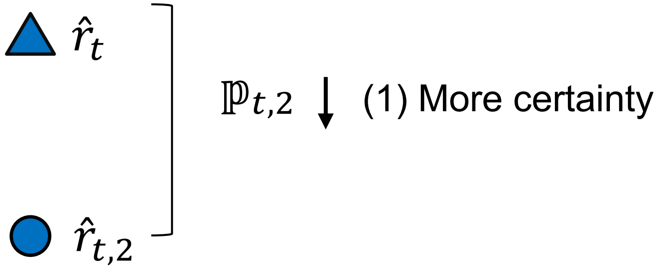


$\hat{r}_t$

$\hat{r}_{t,1}$

$\mathbb{p}_{t,1}$ ↑ (1) Less certainty

Arm 1



$\hat{r}_t$

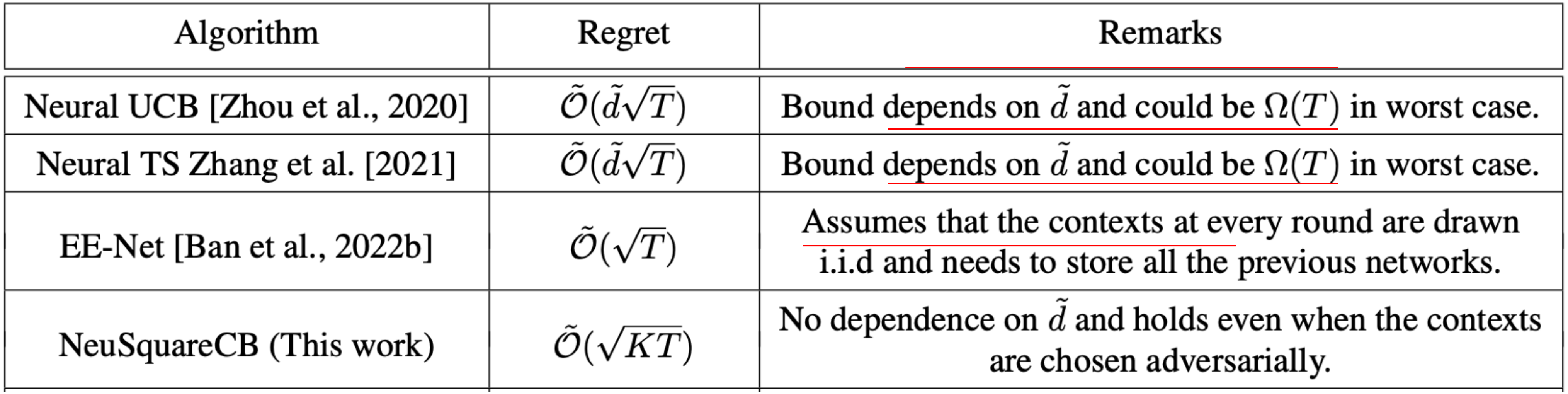
$\hat{r}_{t,2}$

$\mathbb{p}_{t,2}$ ↓ (1) More certainty

Arm 2

1. Deb, Rohan, Yikun Ban, Shiliang Zuo, Jingrui He, and Arindam Banerjee. "Contextual bandits with online neural regression." ICLR 2024.

Neural SquareCB: Inverse Gap Strategy



Algorithm	Regret	Remarks
Neural UCB [Zhou et al., 2020]	$\tilde{O}(\tilde{d}\sqrt{T})$	Bound depends on $\tilde{d}$ and could be $\Omega(T)$ in worst case.
Neural TS Zhang et al. [2021]	$\tilde{O}(\tilde{d}\sqrt{T})$	Bound depends on $\tilde{d}$ and could be $\Omega(T)$ in worst case.
EE-Net [Ban et al., 2022b]	$\tilde{O}(\sqrt{T})$	Assumes that the contexts at every round are drawn i.i.d and needs to store all the previous networks.
NeuSquareCB (This work)	$\tilde{O}(\sqrt{KT})$	No dependence on $\tilde{d}$ and holds even when the contexts are chosen adversarially.

- ➢ Remove dependence of effective dimension.
- ➢ Minimize dependence on Neural Tangent Kernel.

1. Deb, Rohan, Yikun Ban, Shiliang Zuo, Jingrui He, and Arindam Banerjee. "Contextual bandits with online neural regression." ICLR 2024.

Takeaways

Fundamental Exploration

- Neural UCB [1] -- An Extension of LinUCB to NTK Space
- Neural TS [2] -- An Extension of LinTS to NTK Space
- EE-Net [3] -- Another Neural Network for Exploration

Efficient Exploration

- Neural Linear UCB [4] -- LinUCB with Neural Representation
- Neural Network with Perturbed Reward [5] -- Implicit Exploration by Perturbing Rewards
- Neural Square CB[6] -- Exploration using Inverse Weight Gap Strategy

1. Zhou, Dongruo, Lihong Li, and Quanquan Gu. "Neural contextual bandits with ucb-based exploration." ICML 2020.
2. Zhang, Weitong, Dongruo Zhou, Lihong Li, and Quanquan Gu. "Neural thompson sampling." ICLR 2021.
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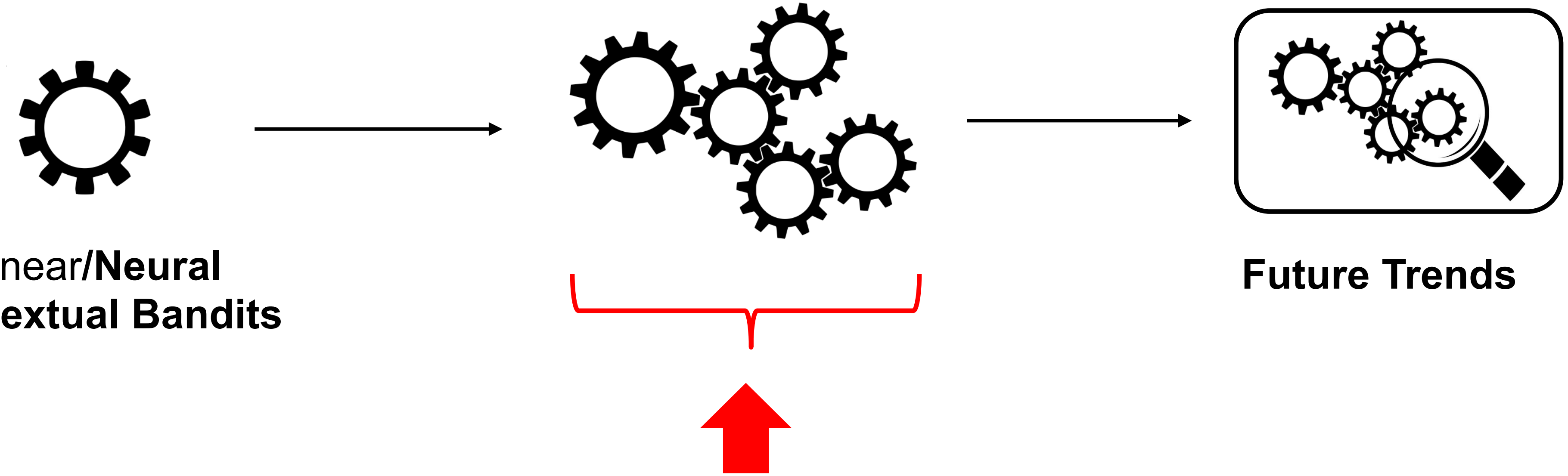
6. Deb, Rohan, Yikun Ban, Shiliang Zuo, Jingrui He, and Arindam Banerjee. "Contextual bandits with online neural regression." ICLR 2024.

Tutorial Roadmap



Collaborative Bandits

Linear/**Neural Contextual Bandits**

Future Trends

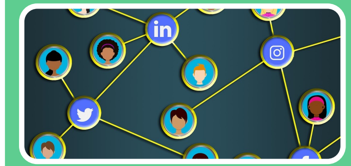
Collaborative Bandits

Introduction

- Background & Motivations
- Challenges

Online Clustering of Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits



Graph Bandit Learning with Collaboration

- User side: Graph Neural Bandits
- Arm side: Neural Bandit with Arm Group Graph
- Other Scenarios: Bandit Learning with Graph Feedback & Online Graph Classification with Neural Bandit

Bandits for Combo Recommendation

- Multi-facet Contextual Bandits

Collaborative Contextual Bandits: Background & Motivation

- Conventional approaches, e.g., **collaborative and content-based filtering:**

		📖	🛍	🎧	🎮
A	👤	✓	✗	✓	✓
B	👤		✓	✗	✗
C	👤	✓	✓	✗	
D	👤	✗		✓	
E	👤	✓	✓	?	✗

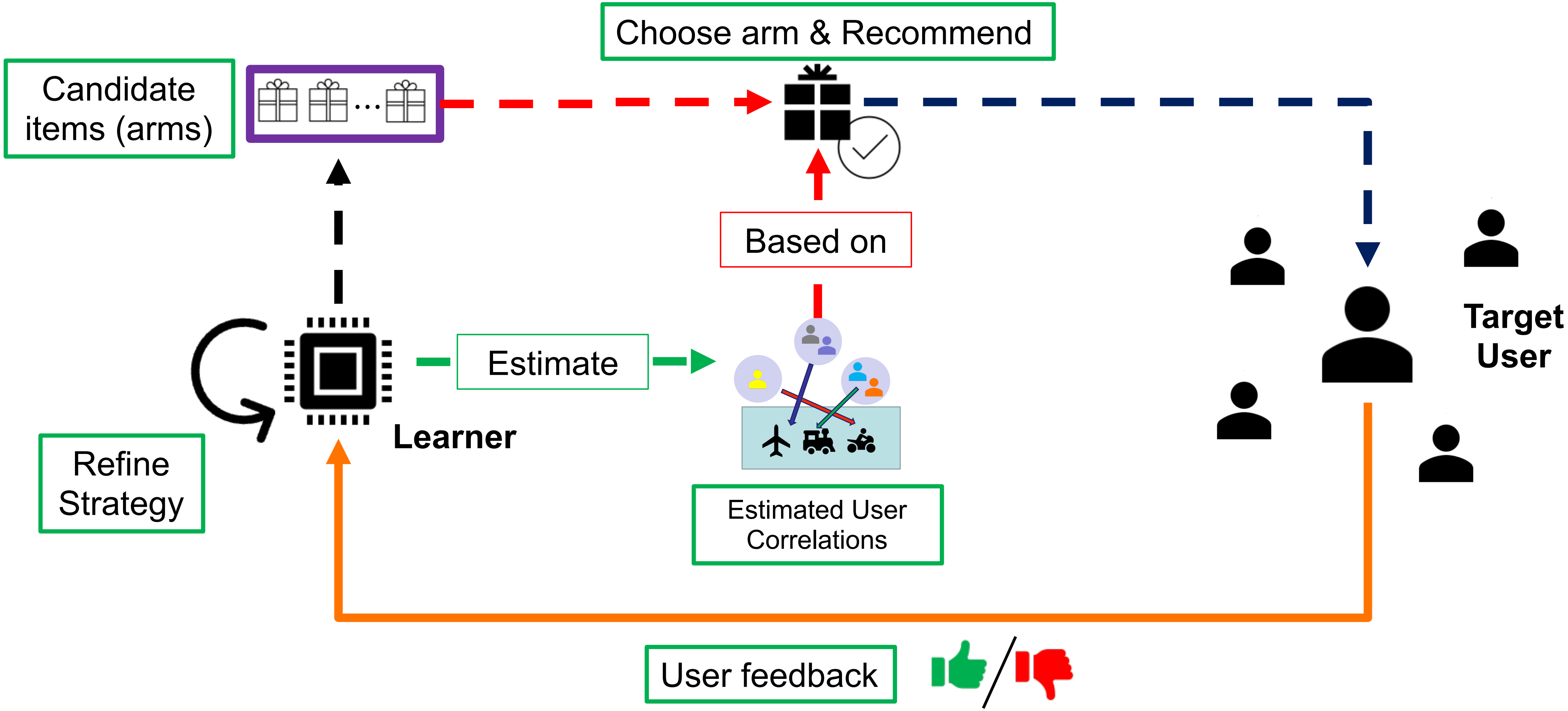
(InCube Group)

Challenges:

- **Cold-start** problem (Lack of history data);
- **Rapid change** of recommendation content and user interests.
- Dilemma of **Exploitation** and **Exploration.**

Collaborative Contextual Bandits: Background & Motivation

❑ **Online recommendation scenario** (in each round)**:**



Choose arm & Recommend

Candidate items (arms)

Based on

Estimate

Learner

Refine Strategy

Estimated User Correlations

Target User

User feedback

---

1. Lihong Li, et al. 2010. A contextual-bandit approach to personalized news article recommendation. In WWW. 661–670.
2. Claudio Gentile, et al. 2014. Online clustering of bandits. In ICML. 757–765.

Collaborative Contextual Bandits: Background & Motivation

☐ **The dilemma of exploitation and exploration:**

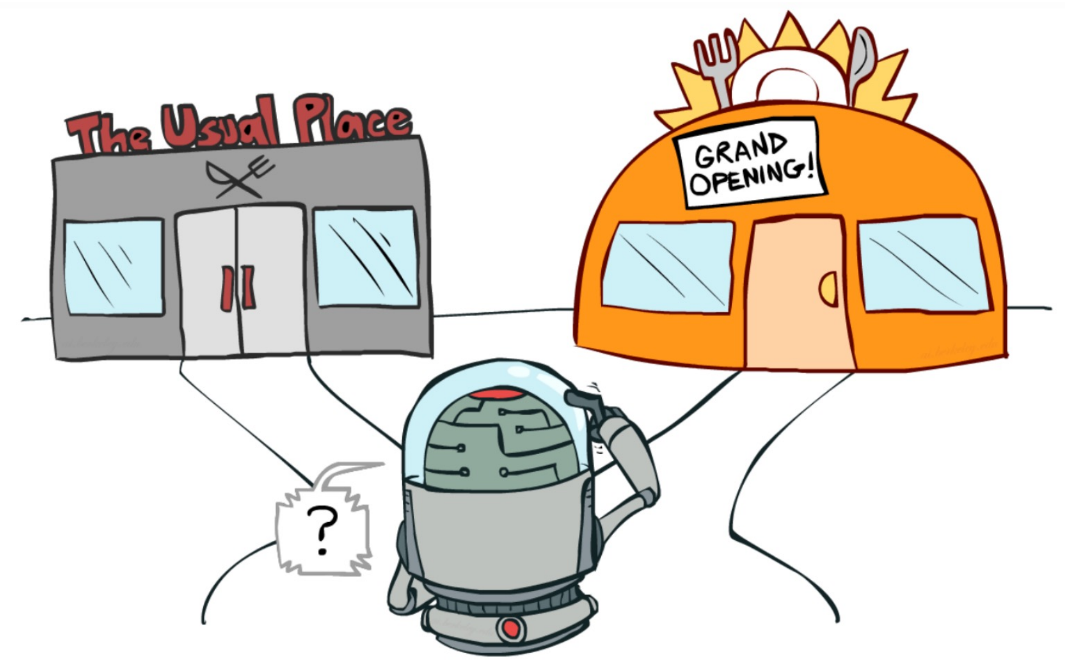
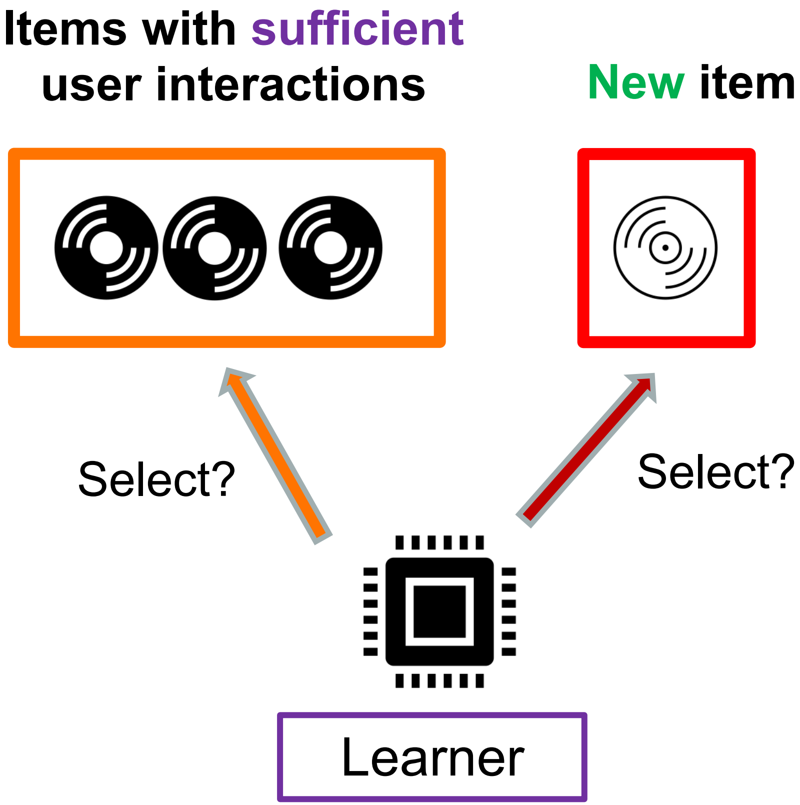


Figure: UC Berkeley CS 188, Introduction to Artificial Intelligence



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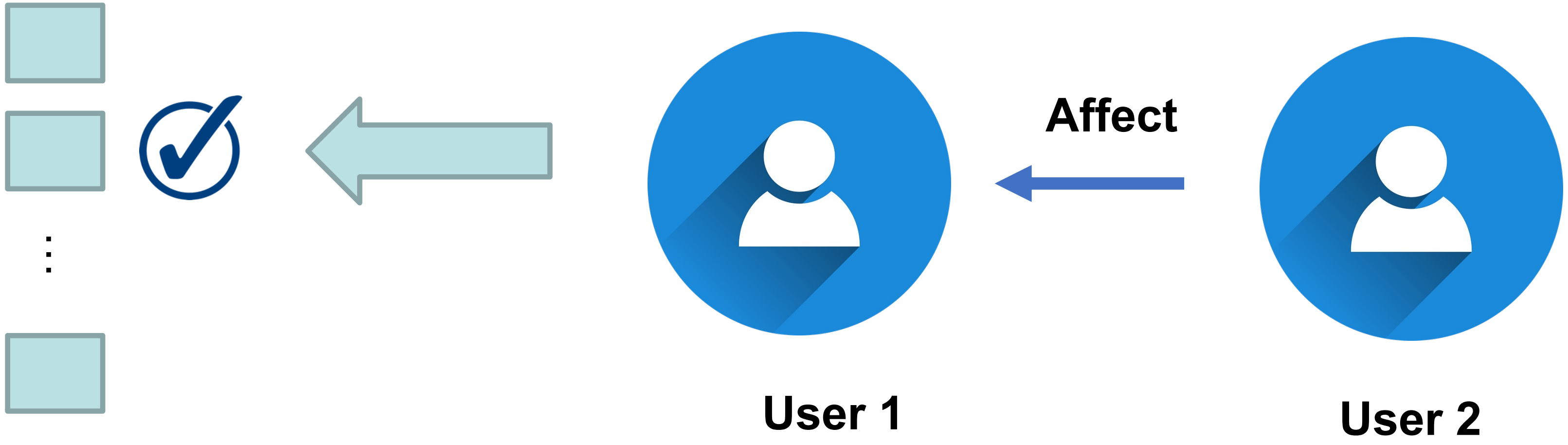
Collaborative Contextual Bandits: Background & Motivation

❑ **One user's decision is affected by other users.**



❑ **Motivations**: Utilizing the **mutual influence / user collaborative effects** can

- Improve **recommendation quality**.
- Alleviate the **interaction scarcity** issue in terms of individual users.
- Rapidly adapt to **new users / items** based on interactions with other users.

---

1. Lihong Li, et al. 2010. A contextual-bandit approach to personalized news article recommendation. In WWW. 661–670.
2. Claudio Gentile, et al. 2014. Online clustering of bandits. In ICML. 757–765.

Collaborative Contextual Bandits: Challenges

☐ ***Challenge* #1**: How to **formally model user collaborations?**

- User clusters [1, 2, 3, 4, 5, 6, 7], graphs with user nodes [10], etc.

☐ ***Challenge* #2**: How to **discover user correlations?**

- Leveraging the **known** user correlation information from the environment [8, 9];
- User clustering based on their past interactions [2,3,4,5,7], exploitation-exploration graph construction [10].

☐ ***Challenge* #3**: How to **utilize user correlation** to **improve recommendation quality?**

- Combination of linear estimations [1, 2, 3, 4, 5, 6], gradient-based meta-learning [7], graph neural networks [10], etc.

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1. Gentile et. al., Online clustering of bandits. ICML 2014.
2. Li et. al., Improved algorithm on online clustering of bandits. IJCAI 2019.
3. Nguyen et. al., Dynamic clustering of contextual multi-armed bandits. CIKM 2014.
4. Gentile et. al., On context-dependent clustering of bandits. ICML 2017.
5. Ban et. al., Local clustering in contextual multi-armed bandits. WWW 2021.
6. Li et. al., Collaborative filtering bandits. SIGIR 2016.
7. Ban et. al., Meta clustering of neural bandits. In submission.
8. Nicolo Cesa-Bianch et. al., A gang of bandits. NIPS 2013.
9. Wu et. al., Contextual bandits in a collaborative environment. SIGIR 2016.
10. Qi et. al., Graph neural bandits. KDD 2023.

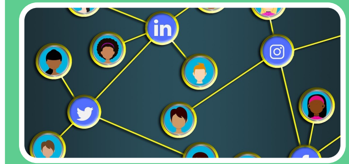
Roadmap

Introduction

- Background & Motivations
- Challenges

Online Clustering of Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits



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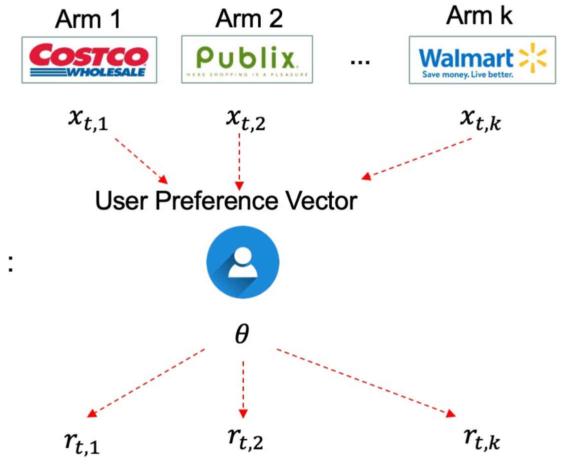
Online Clustering of Bandits

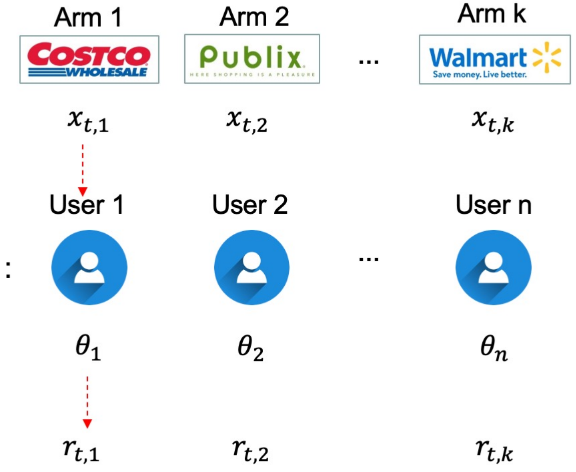
☐ Two problem settings in standard MAB algorithms:

- Ignore user heterogeneity



(1) Joint Modeling

- Ignore user correlations



(2) Disjoint Modeling

☐ For **trade-off** between user heterogeneity and user correlations:

- ○ Objective #1: **Identify user clusters** in MAB;
- ○ Objective #2: **Exploit the user clusters** to improve the recommendation.

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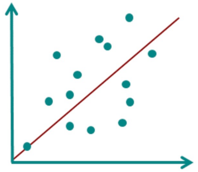
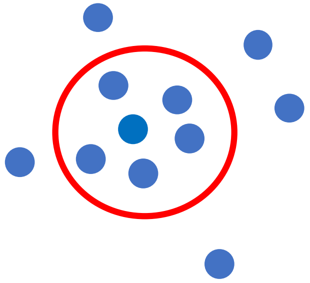
1. Gentile et. al., Online clustering of bandits. ICML 2014.
2. Li et. al., Improved algorithm on online clustering of bandits. IJCAI 2019.
3. Nguyen et. al., Dynamic clustering of contextual multi-armed bandits. CIKM 2014.
4. Gentile et. al., On context-dependent clustering of bandits. ICML 2017.
5. Ban et. al., Local clustering in contextual multi-armed bandits. WWW 2021.
6. Li et. al., Collaborative filtering bandits. SIGIR 2016.

Online Clustering of Linear Bandits

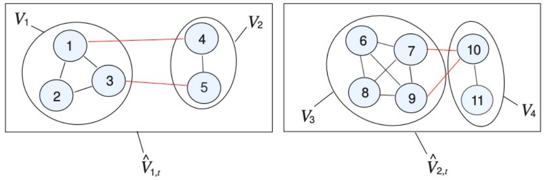
➢ **Clustering of <u>Linear</u> Bandits:**

- ❑ Under **linear** stochastic contextual bandit settings: $r = \langle \theta_u , x \rangle + \eta$.

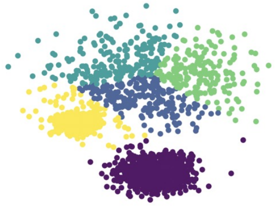




- ❑ User **correlation intensity** between u, u' is measured by $\left\|\theta_u - \theta_{u'}\right\|_2$.
  1. User clusters with **identical preferences** [1, 2, 3, 4, 5] ($\forall u, u' \in \mathcal{N}$: $\theta_u = \theta_{u'}$).
     - ○ Globle clustering with evolving connected components



  3. **A generalized formulation:** γ-cluster of users [6] ($\forall u, u' \in \mathcal{N}$: $\left\| \theta_u - \theta_{u'} \right\|_2 \leq \gamma$).
     - ○ Seed-based Local clustering



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1. Gentile et. al., Online clustering of bandits. ICML 2014.
2. Li et. al., Improved algorithm on online clustering of bandits. IJCAI 2019.
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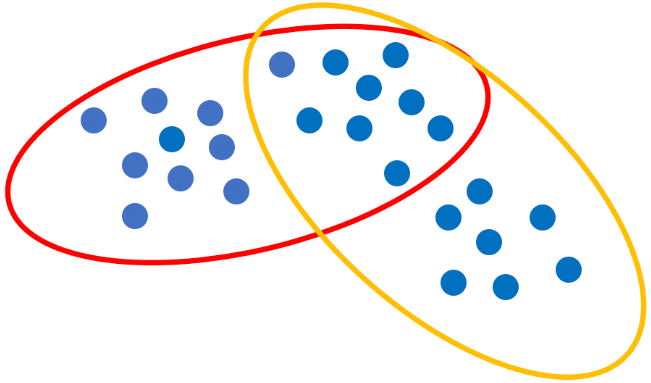
5. Li et. al., Collaborative filtering bandits. SIGIR 2016.
6. Ban et. al., Local clustering in contextual multi-armed bandits. WWW 2021.

LOCB: Motivation and Challenges

- **Challenge 1: When to ensure a set of identified users is a true cluster?**
  - ☐ Cluster: A set of users with similar expected rewards.
  - ☐ Expected rewards of users are unknown.



- **Challenge 2: Can we further reduce the clustering complexity?**
  - ☐ Previous works have clustering complexity $O(n)$.
  - ☐ n is the number of users.

- **Challenge 3: Can we consider and address soft clustering?**
  - ☐ Consider overlapping clusters.
  - ☐ A user is allowed to belong to multiple clusters.

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1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Local Clustering of Linear bandits

- **Characterizing similar users' behaviors:**
  - **Definition (γ-Cluster)**: Given a subset of users $\mathcal{N} \subseteq N$ and a threshold $\gamma > 0$, $\mathcal{N}$ is considered a γ-Cluster if it satisfies: $\forall i, j \in \mathcal{N}, \left\|\theta^i - \theta^j\right\| < \gamma$.

- **Objectives:**
  - **Objective #1**: **Identify clusters** among users, such that the clusters returned by the proposed algorithm are true γ–Clusters with probability at least $1-\delta$.
  - **Objective #2**: Leverage user clusters to improve the quality of recommendation, evaluated by **Regret**.

$$\mathbf{R}_T = \mathbb{E}[\sum_{t=1}^{T} R_t] = \sum_{t=1}^{T} (\underbrace{\boxed{\boldsymbol{\theta}_{i_t}^{\top}\mathbf{x}_t^*}}_{\textbf{Optimal Reward}} - \underbrace{\boxed{\boldsymbol{\theta}_{i_t}^{\top}\mathbf{x}_t}}_{\textbf{Received Reward}})$$

- **Clustering Module** + **Pulling Module**

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1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Clustering Module

- **Identify k clusters, given k seeds in each round**:
  - ☐ **Seed selection**: Randomly choose k users.
  - ☐ **Neighbors**: Two users are neighbors if they belong to the same γ-cluster.
  - ☐ **Potential neighbors**: User i is considered as the potential neighbor of seed user s, when:

$$\|\hat{\boldsymbol{\theta}}_{i,t} - \hat{\boldsymbol{\theta}}_{s,t}\| \leq B_{\boldsymbol{\theta},i}(m_{i,t}, \delta') + B_{\boldsymbol{\theta},s}(m_{s,t}, \delta').$$

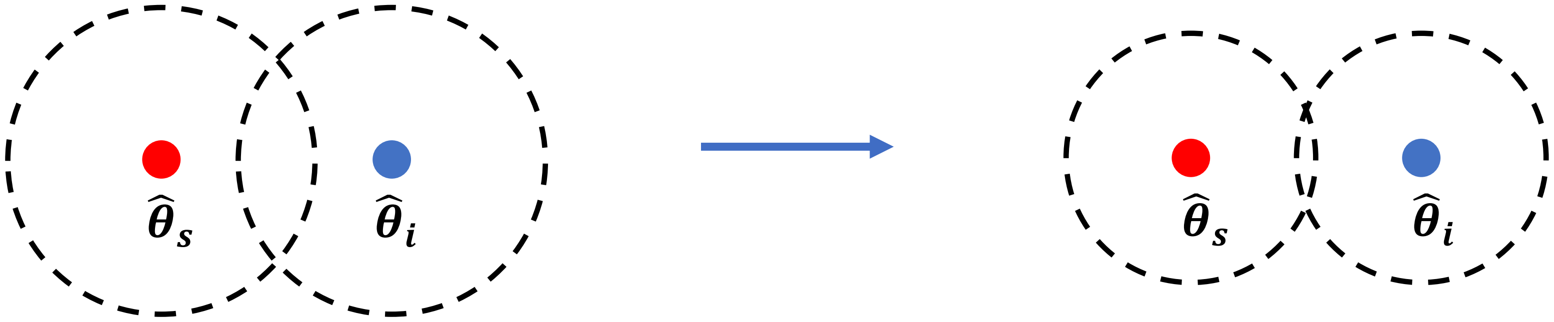
Seed-user parameter **User-specific bound** **Seed-specific bound**

$$B_{\boldsymbol{\theta},i}(m_{i,t}, \delta') = \frac{\sigma\sqrt{2d\log t + 2\log(2/\delta')} + 1}{\sqrt{1 + h(m_{i,t}, H)}},$$

$$h(m_{i,t}, H) = \left(\frac{\lambda m_{i,t}}{4} - 8\log(\frac{m_{i,t}+3}{H}) - 2\sqrt{m_{i,t}\log\left(\frac{m_{i,t}+3}{H}\right)}\right)$$

  - ☐ **Cluster**: Seed user + Its potential neighbors.

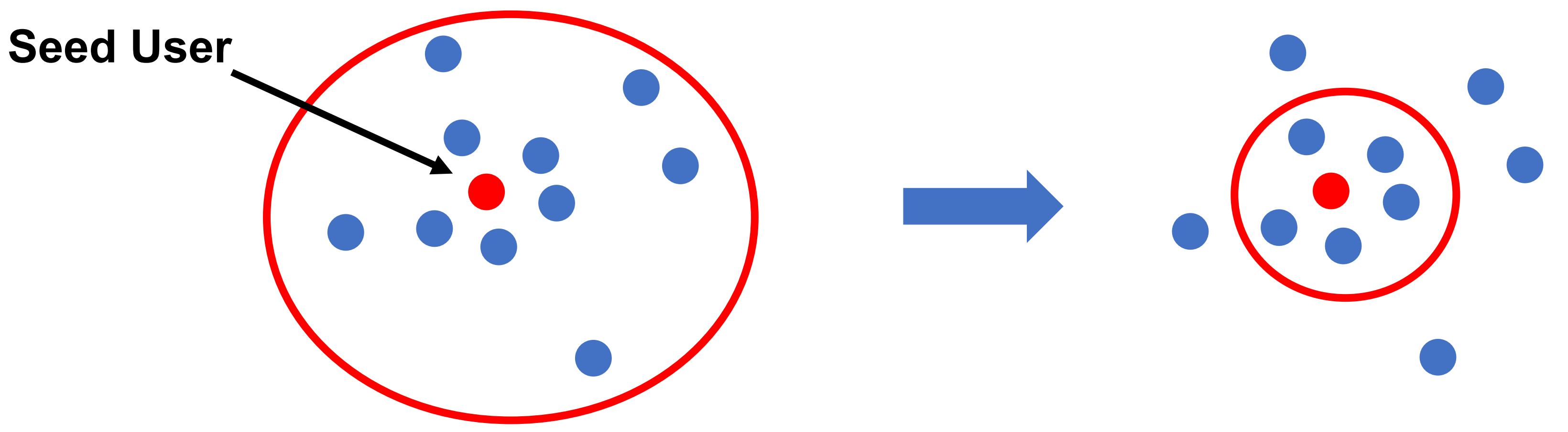
- **User specific bound**: with a high probability, $\|\hat{\theta}_{i,t} - \theta_i\| \leq B_{\theta,i}(m_{i,t}, \delta')$



Potential Neighbors

1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Evolution of Clusters



Cluster s in Round t Cluster s in Round $t + t'$

- **Evolution of neighbors:** $\|\hat{\theta}_{i,t} - \hat{\theta}_{s,t}\| \leq B_{\theta,i}(m_{i,t}, \delta') + B_{\theta,s}(m_{s,t}, \delta')$.

User/seed specific bound is shrinking as more rounds are played for these users.

- **Termination criterion**
  - ❑ Given cluster $\mathcal{N}_{s,t}$, Clustering Module outputs this cluster when

$$\sup\{B_{\theta,i}(m_{i,t}, \delta') : i \in \mathcal{N}_{s,t}\} < \frac{\gamma}{8}$$

1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Pulling Module

- **Individual CB vs. Cluster CB**
  - ❑ Confidence interval for each **cluster**
  - ❑ Confidence interval for each **user**

$$\mathbb{P}\left(\forall t \in [T], |\hat{\boldsymbol{\theta}}_{\mathcal{N}_{s,t}}^{\top}\mathbf{x}_{a,t} - \boldsymbol{\theta}_{\mathcal{N}_{s,t}}^{\top}\mathbf{x}_{a,t}| > \boxed{CB_{r,\mathcal{N}_{s,t}}}\right) < \delta'$$

Cluster CB

$$\mathbb{P}\left(\forall t \in [T], |\hat{\boldsymbol{\theta}}_{i,t}^{\top}\mathbf{x}_{a,t} - \boldsymbol{\theta}_{i}^{\top}\mathbf{x}_{a,t}| > \boxed{CB_{r,i}}\right) < \delta'$$

Individual CB

$$CB_{r,\mathcal{N}_{s,t}} = \frac{1}{|\mathcal{N}_{s,t}|}\sum_{i \in \mathcal{N}_{s,t}} CB_{r,i}$$

- **Pulling Module selects one arm by Cluster UCB**:

Cluster-level exploration

$$\mathbf{x}_t = \arg\max_{\mathbf{x}_{a,t} \in \mathbf{X}_t} \hat{\boldsymbol{\theta}}_{\mathcal{N}_{s,t}}^{\top}\mathbf{x}_{a,t} + CB_{r,\mathcal{N}_{s,t}}$$

Cluster behavior $\hat{\boldsymbol{\theta}}_{\mathcal{N}_{s,t}} = \frac{1}{|\mathcal{N}_{s,t}|}\sum_{i \in \mathcal{N}_{s,t}} \hat{\boldsymbol{\theta}}_{i,t}.$

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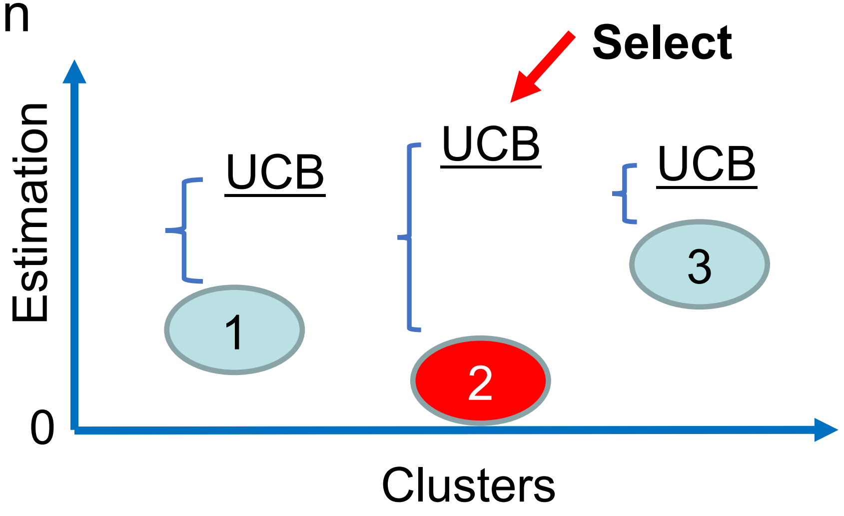
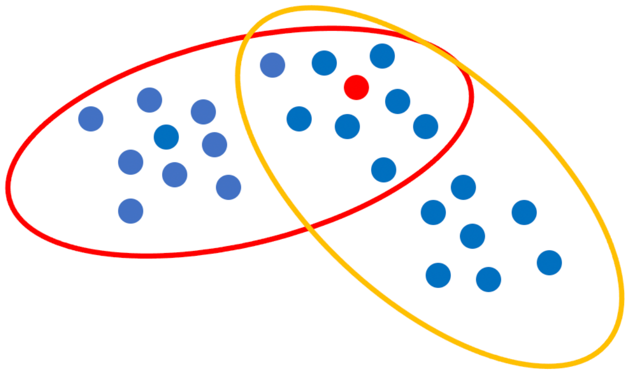
1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Overlapping Clusters

- **A user may belong to multiple overlapping clusters:**
  - Cluster selection





- Pulling Module selects the **cluster with the maximum potential:**

$$\mathbf{x}_t = \arg\max_{\mathbf{x}_{a,t}\in\mathbf{X}_t}\max_{s\in S_t(i_t)}\left(\hat{\theta}^{\intercal}_{\mathcal{N}_{s,t}}\mathbf{x}_{a,t} + CB_{r,\mathcal{N}_{s,t}}\right)$$

Arm set **Cluster set ($O(k)$)**

1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

LOCB: Results

➢ Theoretical analysis：

❑ Correctness ✓

THEOREM 5.1 (CORRECTNESS). *Given a threshold γ and a set of seeds $S \subseteq N$, for each $s \in S$, let $\mathcal{N}_s$ represent the cluster output by LOCB with respect to s. The terminate criterion of Clustering module is defined as:*

$$\sup\{B_{\boldsymbol{\theta},i}(m_{i,t}, \delta') : i \in \mathcal{N}_{s,t}\} < \frac{\gamma}{8}.$$

Then, with probability at least $1 - \delta$, after the Clustering module terminates, for each $s \in S$, it has

$$\forall i, j \in \mathcal{N}_s, \|\boldsymbol{\theta}_i - \boldsymbol{\theta}_j\| < \gamma.$$

❑ Efficiency ✓

THEOREM 5.2. *Suppose each user is evenly served and $m_{i,t} \geq \frac{2\times 32^2}{\lambda^2} \log\left(\frac{2nd}{\delta'}\right) \log\left(\frac{32^2}{\lambda^2} \log\left(\frac{2nd}{\delta'}\right)\right)$ for any $i \in N$. Then, with probability at least $1-\delta$, the number of rounds $\hat{T}$ needed for the Clustering module to terminate is upper bounded by*

$$\hat{T} < \frac{2nd}{C} \log \frac{nd}{C} + \frac{2n}{C}\left(\log(\frac{2^{(d+1)}n}{\delta}) - \frac{\gamma^2 - 256}{512\sigma^2}\right) + n.$$

where $C = \frac{\lambda\gamma^2}{16^3\sigma^2}$.

❑ Effectiveness ✓

THEOREM 5.3. *Suppose that each user is evenly served. Given γ and a set of seeds S, after $T > \hat{T}$ rounds, the accumulated regret of LOCB can be upper bounded as follows:*

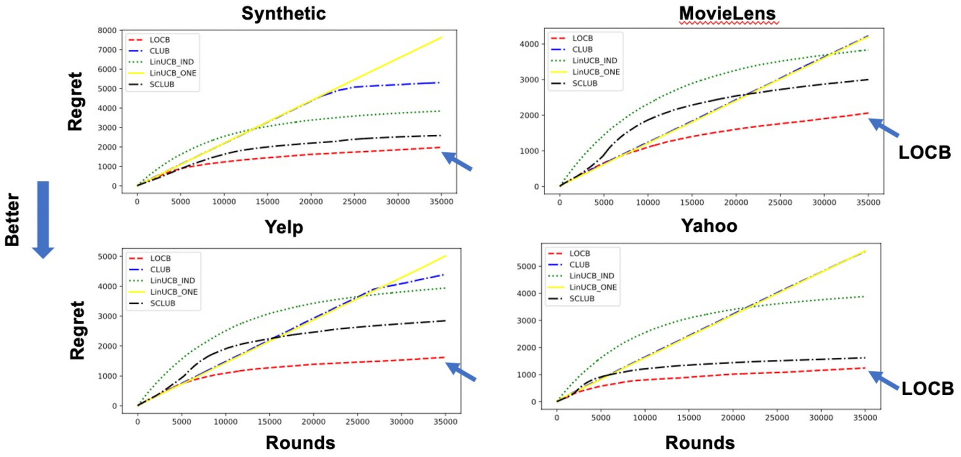
$$\mathbf{R}_T \leq \left[\sqrt{nT} \cdot \sqrt{2d\log(1 + T/dn)} \cdot O\left(\sqrt{d\log\left(T/\delta\right)}\right)\right]$$

$$+ \left(T - O\left(nd\log nd\right)\right)\gamma + O\left(nd\log nd\right) \cdot O\left(\sqrt{d\log\left(Tn/\delta\right)}\right).$$

➢ Evaluations:

❑ Improve performance **up to 12.4%.**

	Synthetic			Yelp				MovieLens			Yahoo		
	F1	Pre	Recall	F1	Pre	Recall		F1	Pre	Recall	F1	Pre	Recall
N-CLUB	0.390	0.246	0.943	0.484	0.334	0.884	N-CLUB	0.417	0.286	0.773	0.454	0.334	0.709
ST-CLUB	0.578	0.549	0.612	0.626	0.593	0.663	ST-CLUB	0.520	0.429	0.663	0.528	0.385	0.841
ST-SCLUB	0.714	0.745	0.687	0.768	0.863	0.693	ST-SCLUB	0.538	0.739	0.424	0.632	0.781	0.532
N-LOCB	0.662	0.618	0.714	0.675	0.620	0.743	N-LOCB	0.472	0.432	0.524	0.615	0.553	0.692
LOCB	**0.880**	**0.913**	**0.856**	**0.879**	**0.908**	**0.853**	LOCB	**0.814**	**0.892**	**0.749**	**0.869**	**0.935**	**0.813**



1. Yikun Ban and Jingrui He. Local Clustering in Contextual Multi-Armed Bandits. WWW 2021.

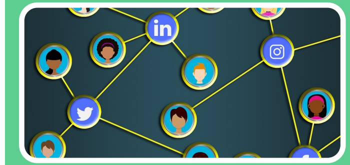
Roadmap

Introduction

- Background & Motivations
- Challenges

Online Clustering of Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits



Graph Bandit Learning with Collaboration

- User side: Graph Neural Bandits
- Arm side: Neural Bandit with Arm Group Graph
- Other Scenarios: Bandit Learning with Graph Feedback & Online Graph Classification with Neural Bandit

Bandits for Combo Recommendation

- Multi-facet Contextual Bandits

Online Clustering of Neural Bandits

- **Challenge 1: How to efficiently determining a user's relative group?**
  - ☐ User relative group: A set of users with **same expected rewards on a specific item (arm)**.
  - ☐ Expected rewards of users are unknown. The mapping function $h(x)$ can be linear or non-linear.

- **Challenge 2: Effective parametric representation of dynamic clusters?**
  - ☐ Introducing **meta-learner** capable of representing and swiftly adapting to evolving user clusters.
  - ☐ Enabling the rapid acquisition of nonlinear cluster representations.

- **Challenge 3: Balancing exploitation and exploration?**
  - ☐ A novel UCB-type exploration strategy.
  - ☐ Taking both user-side and meta-side information into account.

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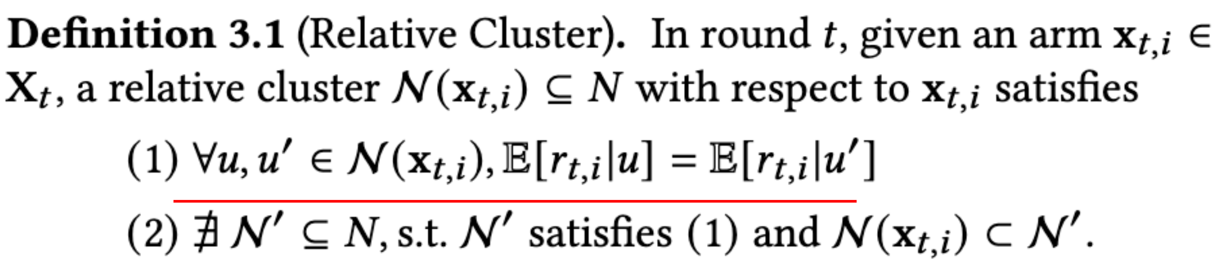
1. Yikun Ban et. al., Meta Clustering of Neural Bandits. In submission.

M-CNB: Meta Clustering of Neural Bandits

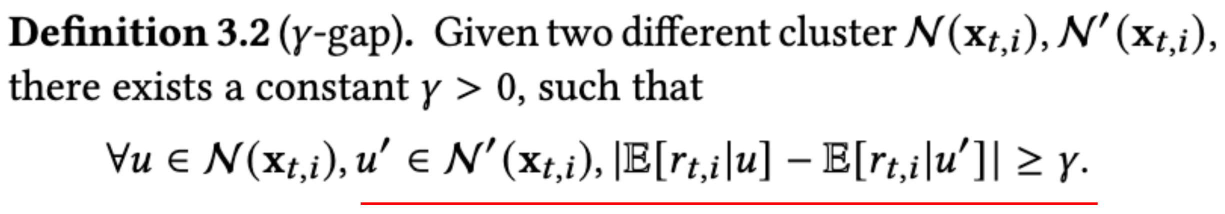
- **Characterizing user clusters without linear assumptions:**



Definition 3.1 (Relative Cluster). In round t, given an arm $\mathbf{x}_{t,i} \in \mathbf{X}_t$, a relative cluster $\mathcal{N}(\mathbf{x}_{t,i}) \subseteq N$ with respect to $\mathbf{x}_{t,i}$ satisfies

(1) $\forall u, u' \in \mathcal{N}(\mathbf{x}_{t,i}), \mathbb{E}[r_{t,i}|u] = \mathbb{E}[r_{t,i}|u']$

(2) $\nexists \mathcal{N}' \subseteq N$, s.t. $\mathcal{N}'$ satisfies (1) and $\mathcal{N}(\mathbf{x}_{t,i}) \subset \mathcal{N}'$.



Definition 3.2 (γ-gap). Given two different cluster $\mathcal{N}(\mathbf{x}_{t,i}), \mathcal{N}'(\mathbf{x}_{t,i})$, there exists a constant $\gamma > 0$, such that

$$\forall u \in \mathcal{N}(\mathbf{x}_{t,i}), u' \in \mathcal{N}'(\mathbf{x}_{t,i}), |\mathbb{E}[r_{t,i}|u] - \mathbb{E}[r_{t,i}|u']| \geq \gamma.$$

- **Objectives:**
  - ❑ Objective #1: **Identify clusters** among users, such that the clusters returned by the proposed algorithm are accurate user clusters.
  - ❑ Objective #2: Leverage user correlations to improve the quality of recommendation, evaluated by **Pseudo Regret**.

$$\mathbf{R}_T = \sum_{t=1}^{T} \mathbb{E}[r_t^* - r_t \mid u_t, \mathbf{X}_t],$$

Optimal Reward **Received Reward**

$$\mathbb{E}[r_t^*|u_t, \mathbf{X}_t] = \max_{\mathbf{x}_{t,i} \in \mathbf{X}_t} h_{u_t}(\mathbf{x}_{t,i})$$

General reward function

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1. Yikun Ban et. al., Meta Clustering of Neural Bandits. In submission.

M-CNB: Clustering Module

- **Identify relative cluster for target user $u_t \in N$:**
  - **Arm-specific**: Different arms can induce distinct user clusters.
  - **User models**: Each user $u \in N$ is assigned with their own user models $f(\cdot\,;\theta^u)$.
  - **Potential neighbors**: User u is the potential neighbor of target user u_t, when:

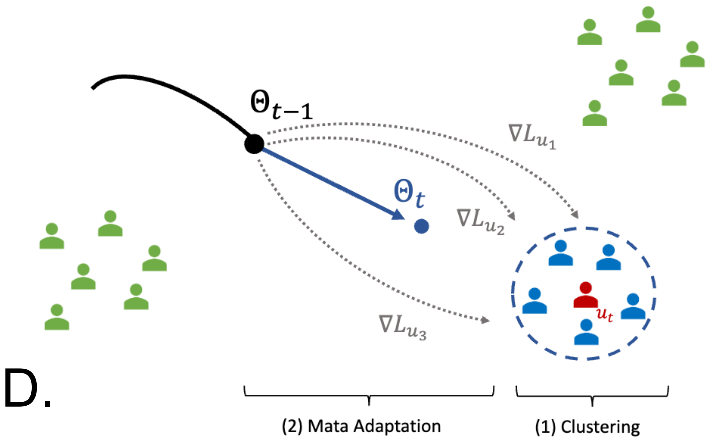
$$\widehat{\mathcal{N}}_{u_t}(\mathbf{x}_{t,i}) = \left\{u \in N \;\middle|\; |f(\mathbf{x}_{t,i};\theta^{u}_{t-1}) - f(\mathbf{x}_{t,i};\theta^{u_t}_{t-1})| \le \frac{\nu-1}{\nu}\gamma\right\}.$$

Preference est. for other users

Preference est. for target user

Tunable distance threshold

- **Meta-adaptation:** Adapting to estimated user clusters.
  - Randomly draw a few samples from the historical data of detected cluster $\{\mathcal{T}^{u}_{t-1}\}_{u\in\widehat{\mathcal{N}}_{u_t}(\mathrm{x}_{i,t})}$.
  - The meta-model $f(\cdot\,;\Theta)$ is adapted through a few steps of SGD.



1. Yikun Ban et. al., Meta Clustering of Neural Bandits. In submission.

M-CNB: Pulling Module

- **Informative UCB for reward estimation:**

Meta-Model Error

$$\sum_{t=1}^{T} \mathbb{E}_{r_t|\mathbf{x}_t}\left[|f(\mathbf{x}_t;\Theta_t) - r_t| \mid u_t\right]$$
$$\le \sum_{t=1}^{T} \underbrace{\frac{O(\boxed{\|\nabla_\Theta f(\mathbf{x}_t;\Theta_t) - \nabla_\theta f(\mathbf{x}_t;\theta_0^{u_t})\|_2})}{m^{1/4}}}_{\text{Meta-side info}} + \sum_{u\in N} \mu_T^u \left[\underbrace{O\left(\sqrt{\frac{S+1}{2\mu_T^u}}\right) + \sqrt{\frac{2\log(1/\delta)}{\mu_T^u}}}_{\text{User-side info}}\right],$$

Gradient Discrepancy between User Model and the Meta-Model

User-side Upper Bound based on Service Frequency

- **Pulling Module selects one arm by Cluster UCB**:

$$\mathbf{x}_t = \arg_{\mathbf{x}_{t,i}\in\mathbf{X}_t} \max \mathbf{U}_{t,i}$$

$$\mathbf{U}_{t,i} = \boxed{f(\mathbf{x}_{t,i};\Theta_{t,i})} + \boxed{\frac{\|\nabla_\Theta f(\mathbf{x}_{t,i};\Theta_{t,i}) - \nabla_\theta f(\mathbf{x}_{t,i};\theta_0^{u_t})\|_2}{m^{1/4}} + \sqrt{\frac{S+1}{2\mu_t^u}} + \sqrt{\frac{2\log(1/\delta)}{\mu_t^u}}}$$

Meta-Model Reward Estimation

UCB

1. Yikun Ban et. al., Meta Clustering of Neural Bandits. In submission.

M-CNB: Theoretical and Empirical Results

- **Theoretical analysis from two aspects**:

❑ Instance-dependent Regret Bound ✓

Theorem 5.1. *Given the number of rounds* T *and* γ, *for any* $\delta \in (0,1), R > 0$, *suppose* $m \geq \widetilde{\Omega}(poly(T, L, R) \cdot Kn\log(1/\delta))$, $\eta_1 = \eta_2 = \frac{R^2}{\sqrt{m}}$, *and* $\mathbb{E}[|\mathcal{N}_{u_t}(\mathbf{x}_t)|] = \frac{n}{q}, t \in [T]$. *Then, with probability at least* $1-\delta$ *over the initialization, Algorithm 1 achieves the following regret upper bound:*

$$\mathbf{R}_T \leq \sqrt{qT \cdot S^*_{TK} + O(1)} + O(\sqrt{2qT\log(O(1)/\delta)}).$$

where $S^*_{TK} = \inf_{\theta \in B(\theta_0, R)} \sum_{t=1}^{TK} \mathcal{L}_t(\theta)$.

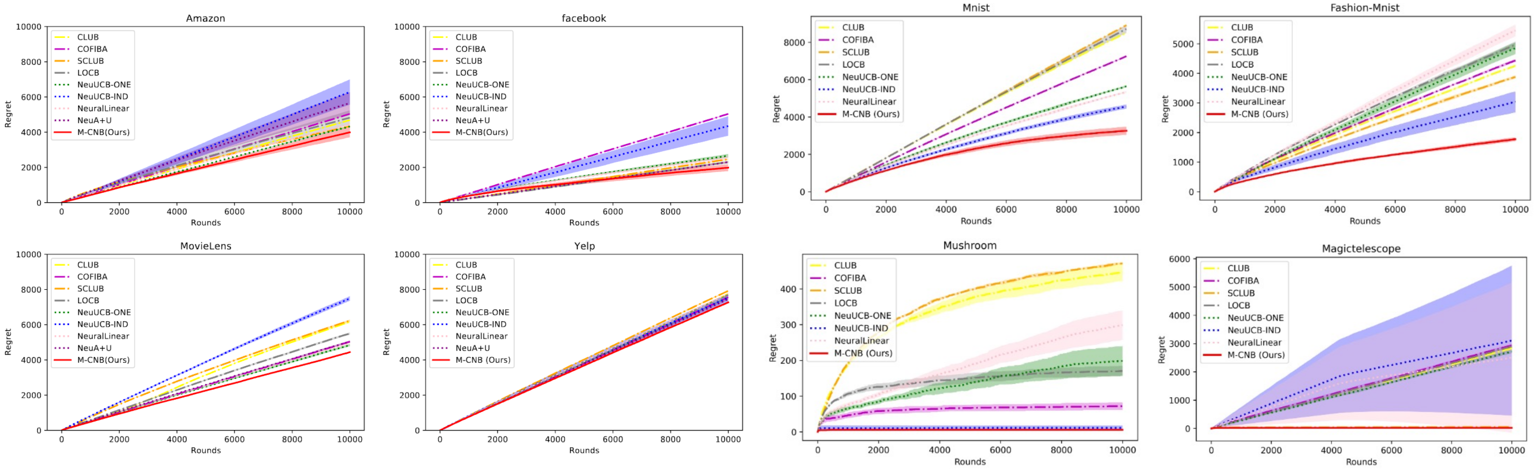
❑ NTK-regression based Regret Bound ✓

Lemma 5.3. *Suppose Assumption 5.1 and conditions in Theorem 5.1 holds where* $m \geq \widetilde{\Omega}(poly(T, L) \cdot Kn\lambda_0^{-1}\log(1/\delta))$. *With probability at least* $1-\delta$ *over the initialization, there exists* $\theta' \in B(\theta_0, \widetilde{\Omega}(T^{3/2}))$, *such that*

$$\mathbb{E}[S^*_{TK}] \leq \mathbb{E}[\sum_{t=1}^{TK} \mathcal{L}_t(\theta')] \leq \widetilde{O}\left(\sqrt{\widetilde{d}} + S\right)^2 \cdot \widetilde{d}.$$

- **Evaluations**:

❑ M-CNB (red curve) outperforms baselines, for both recommendation and classification data sets.



1. Yikun Ban et. al., Meta Clustering of Neural Bandits. In submission.

Online Clustering of Bandits

- ☐ Motivations: We need to **estimate user correlations on the fly**, during online recommendation.
  - ☐ **Clustering of Linear Bandits** [1, 2, 3, 4, 5, 6]**:**
    - ○ Under **linear** stochastic contextual bandit settings: $r = \langle \theta_u, x \rangle + \eta$.
    - ○ User **correlation intensity** between u, u' is measured by $\left\| \theta_u - \theta_{u'} \right\|_2$.
    - ○ Adopt *combination of linear estimators* for reward estimation & exploration.
  - ☐ **Clustering of Neural Bandits** [7]**:**
    - ○ Under **neural** stochastic contextual bandit settings: $r = h_u(x) + \eta$.
    - ○ User clusters with identical preferences ($\forall u, u' \in \mathcal{N}, x \in \mathbb{R}^d$: $h_u(x) = h_{u'}(x)$).
    - ○ Utilizing *gradient-based Meta-Learning* for reward estimation & exploration.

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1. Gentile et. al., Online clustering of bandits. ICML 2014.
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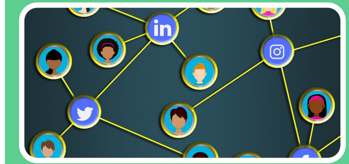


Graph Bandit Learning with Collaboration

- User side: Graph Neural Bandits
- Arm side: Neural Bandit with Arm Group Graph
- Other Scenarios: Bandit Learning with Graph Feedback & Online Graph Classification with Neural Bandit

Application in Recommender Systems

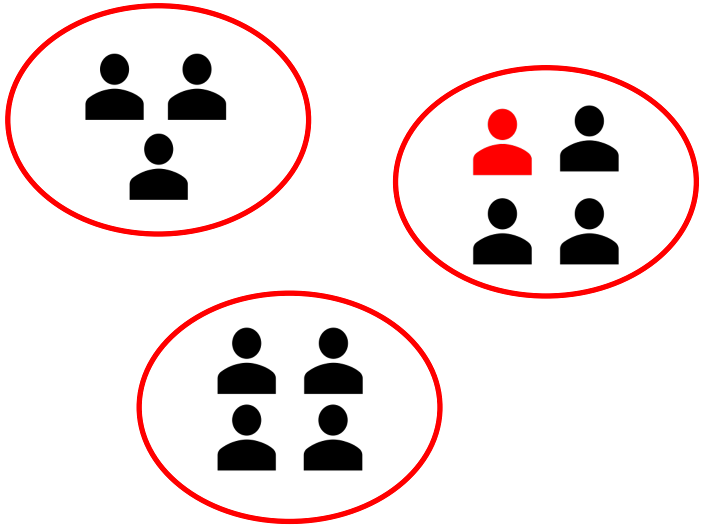
- Multi-facet Personalized Recommendation

Collaborative Exploration: Graph Bandits Learning

👤 User (Bandit)

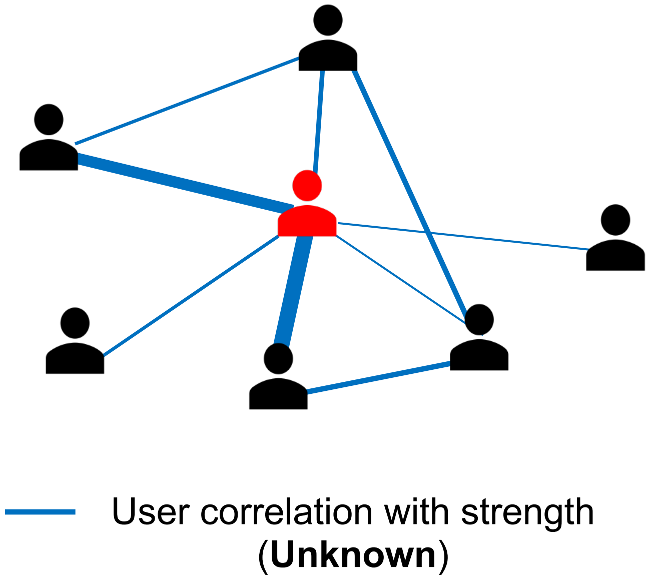
Clustering of Bandits [1,2]

- **Coarse-grained** user correlations:
  - Users within the same cluster share **identical preferences**.
  - **Contribute equally** to serving user.



Graph Bandits Learning [3]

- **Fine-grained** user correlations:
  - Heterogeneity of users is preserved.
  - **Contribute differently** to serving user.



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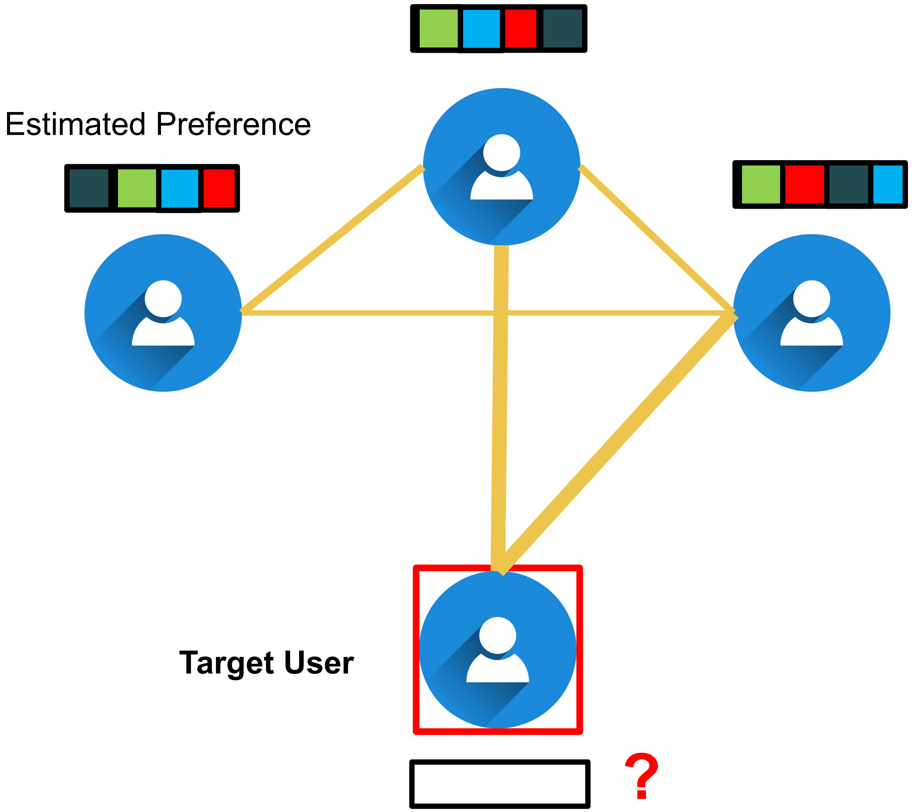
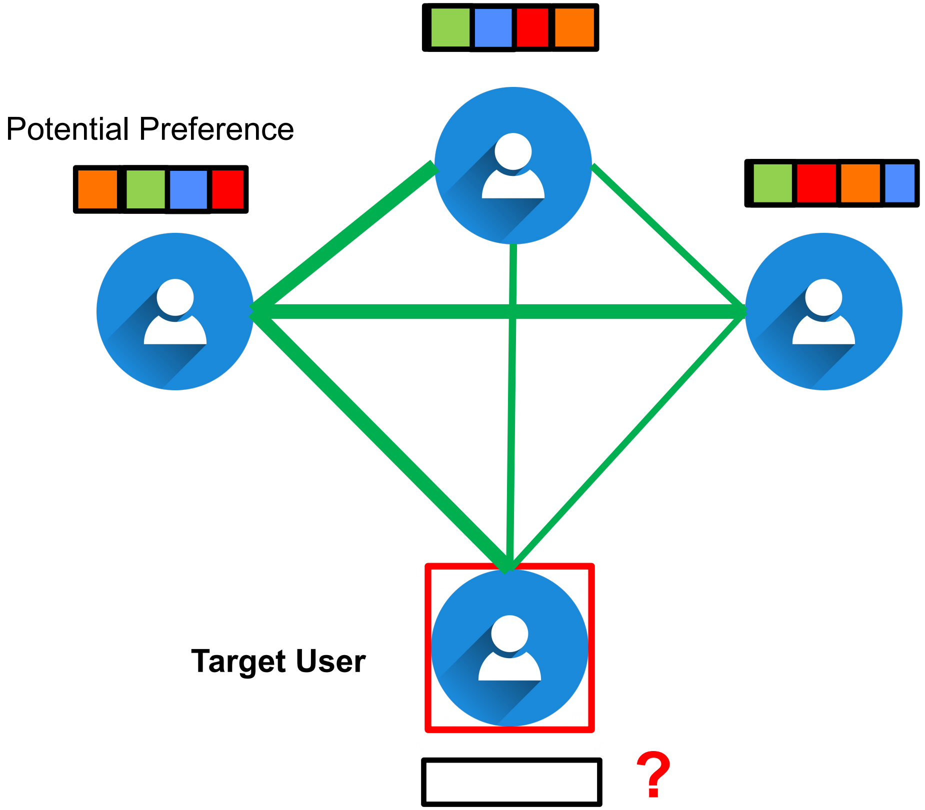
1. Claudio Gentile, et al. 2014. Online clustering of bandits. In ICML. 757–765.
2. Shuai Li, et al. 2019. Improved Algorithm on Online Clustering of Bandits. In IJCAI. 2923–2929.
3. Y. Qi, Y. Ban*, and J. He. Graph neural bandits. KDD 2023.

GNB: Exploitation and Exploration Graphs



User Exploitation Graph

— **Correlation w.r.t. Exploitation**



User Exploration Graph

— **Correlation w.r.t. Exploration**

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Problem Definition

- **For each round** $t \in [T]$ **:**
  - ☐ Receive a target user $u_t \in \mathcal{U}$, and candidate arms (items) $\mathcal{X}_t$.
    - ○ $\mathcal{X}_t = \{\boldsymbol{x}_{i,t} \in \mathbb{R}^d,\ \ (\text{e.g.,}$ 💿 $)\}_{i \in [a]}$
  - ☐ Reward $r_{i,t} = h\left(\mathcal{G}_{i,t}^{(1),*}, u_t, x_{i,t}\right) + \underline{\epsilon_{i,t}}.$

    Zero-mean noise
  - ☐ Learner **selects** arm $x_t \in \mathcal{X}_t$ as the recommendation.

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1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Problem Definition

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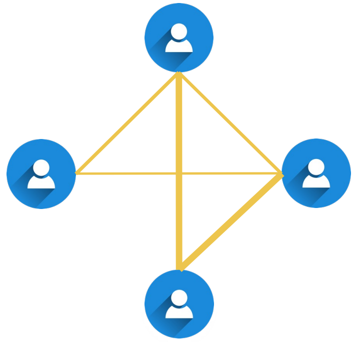
  - ☐ Reward $r_{i,t} = h\left(\underline{\mathcal{G}_{i,t}^{(1),*}}, u_t, \ x_{i,t}\right) + \epsilon_{i,t}$.
  - ☐ Learner **selects** arm $x_t \in \mathcal{X}_t$ as the recommendation.

- **Definition: User Correlation (Exploitation) Graph**
  - ☐ Given arm $\boldsymbol{x}_{i,t}$, **unknown** user exploitation graph $\mathcal{G}_{i,t}^{(1),*} = \left(\mathcal{U}, E, W_{i,t}^{(1),*}\right)$:
    - $\mathcal{U}$: set of nodes (**users**)
    - $E = \{e(u, u')\}_{u,u' \in \mathcal{U}}$: set of edges
    - $W_{i,t}^{(1),*}$: set of edge weights

$$W_{i,t}^{(1),*} = \boldsymbol{\Psi}^{(1)}\left(\mathbb{E}[\boldsymbol{r}_{i,t} \mid \boldsymbol{u}_1, \boldsymbol{x}_{i,t}], \mathbb{E}[\boldsymbol{r}_{i,t} \mid \boldsymbol{u}_2, \boldsymbol{x}_{i,t}]\right)$$



User correlations w.r.t. the expected reward (**Exploitation Graph**)

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1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: User Exploration Graph

- **Definition: User Exploration Graph**
  - ☐ For arm $\boldsymbol{x}_{i,t}$, **unknown** user exploration graph

$$\mathcal{G}_{i,t}^{(2),*} = (\mathcal{U}, E, \underline{W_{i,t}^{(2),*}})$$

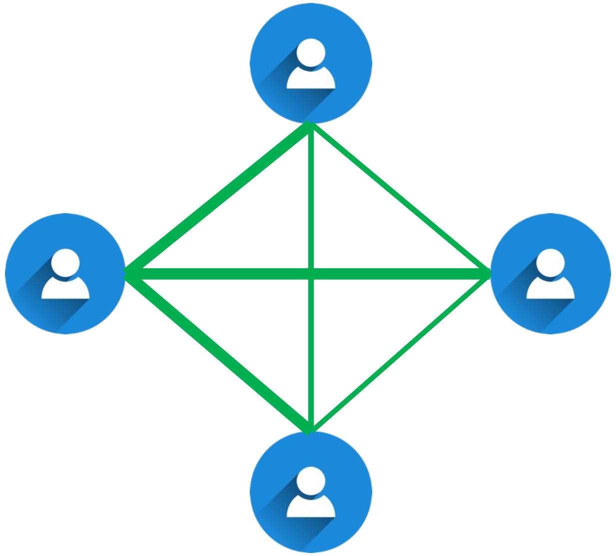
Set of edge weights

  - ☐ For users $u_1, u_2 \in \mathcal{U}$, the corresponding edge weight:
    - $$w_{i,t}^{(2),*}(u_1, u_2) = \underline{\boldsymbol{\Psi}^{(2)}} \Big(\underbrace{\mathbb{E}[\boldsymbol{r}_{i,t} | \boldsymbol{u}_1, \boldsymbol{x}_{i,t}] - \boldsymbol{f}_{\boldsymbol{u}_1}^{(1)}(\boldsymbol{x}_{i,t}), \quad \mathbb{E}[\boldsymbol{r}_{i,t} | \boldsymbol{u}_2, \boldsymbol{x}_{i,t}] - \boldsymbol{f}_{\boldsymbol{u}_2}^{(1)}(\boldsymbol{x}_{i,t})}_{\boxed{\text{Potential Gain}}} \Big)$$

Pre-defined mapping

Potential Gain:

- $\mathbb{E}[\boldsymbol{r} \mid \boldsymbol{u}, \boldsymbol{x}] - f_u^{(1)}(\boldsymbol{x})$
- Measures the uncertainty for the reward estimation



User correlations w.r.t. the Potential Gain (**Exploration Graph**)

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1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.
2. Yikun Ban, et al. EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits. In ICLR 2022.

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  - ☐ Learner **selects** arm $x_t \in \mathcal{X}_t$ as the recommendation.

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    - $w_{i,t}^{(1),*}(u_1, u_2) = \boldsymbol{\Psi}^{(1)}(\mathbb{E}[\boldsymbol{r}_{i,t} \mid \boldsymbol{u}_1, \boldsymbol{x}_{i,t}], \mathbb{E}[\boldsymbol{r}_{i,t} \mid \boldsymbol{u}_2, \boldsymbol{x}_{i,t}])$

- **Objective: Minimizing Pseudo Regret**

Chosen arm reward

$$\mathrm{R(T)} = \sum_{t=1}^{T} \mathbb{E}[r_t^* - r_t]$$

Optimal arm reward

$$\mathbb{E}\,[r_t^*] = \max_{i\in[a]} \mathbb{E}[r_{i,t}]$$

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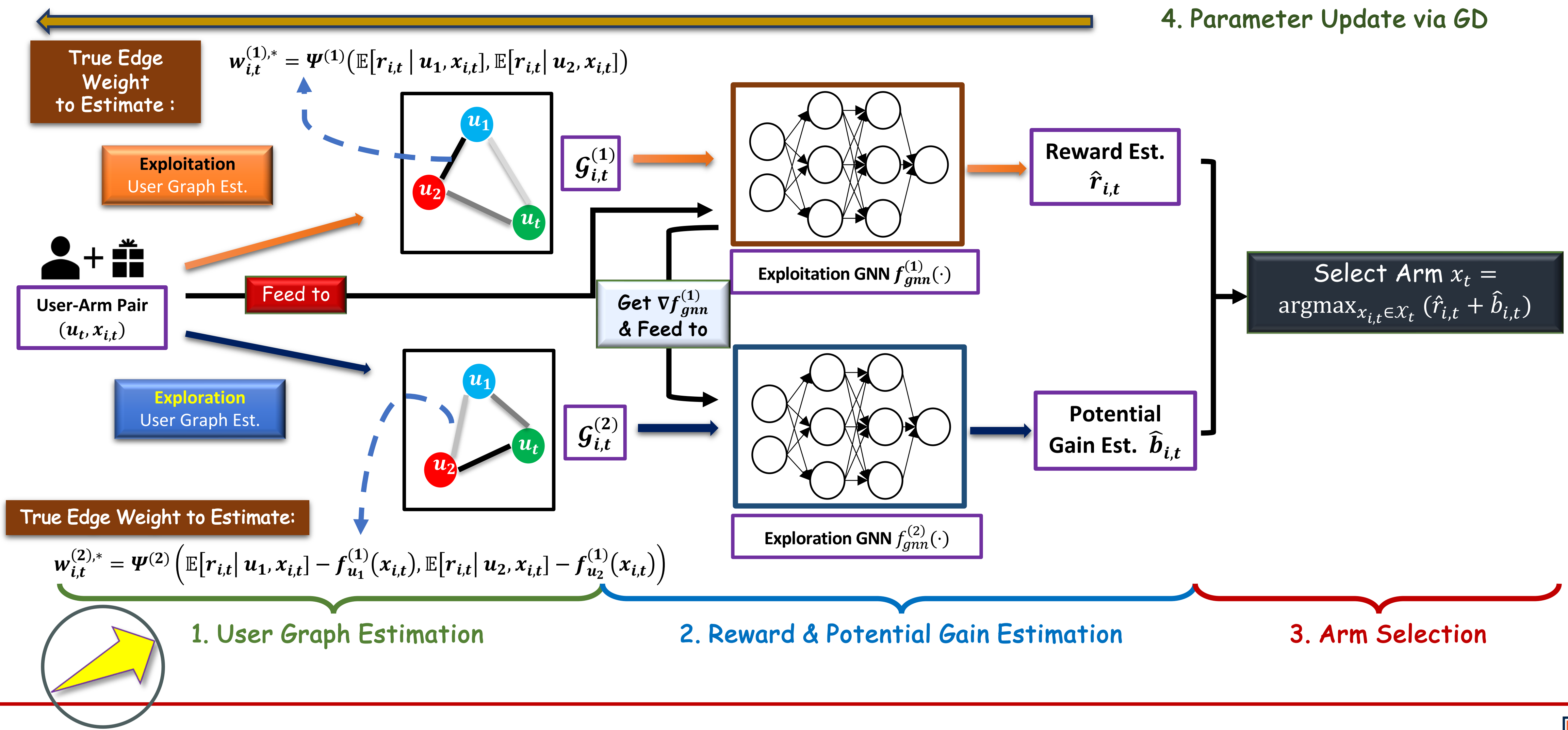
1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Framework Overview



4. Parameter Update via GD

True Edge Weight to Estimate :

$$w_{i,t}^{(1),*} = \Psi^{(1)}(\mathbb{E}[r_{i,t} \mid u_1, x_{i,t}], \mathbb{E}[r_{i,t} \mid u_2, x_{i,t}])$$

Exploitation User Graph Est.

User-Arm Pair $(u_t, x_{i,t})$

Feed to

Exploration User Graph Est.

$\mathcal{G}_{i,t}^{(1)}$

$\mathcal{G}_{i,t}^{(2)}$

Get $\nabla f_{gnn}^{(1)}$ & Feed to

Exploitation GNN $f_{gnn}^{(1)}(\cdot)$

Exploration GNN $f_{gnn}^{(2)}(\cdot)$

Reward Est. $\hat{r}_{i,t}$

Potential Gain Est. $\hat{b}_{i,t}$

Select Arm $x_t = \text{argmax}_{x_{i,t} \in \mathcal{X}_t} (\hat{r}_{i,t} + \hat{b}_{i,t})$

True Edge Weight to Estimate:

$$w_{i,t}^{(2),*} = \Psi^{(2)}\left(\mathbb{E}[r_{i,t} \mid u_1, x_{i,t}] - f_{u_1}^{(1)}(x_{i,t}), \mathbb{E}[r_{i,t} \mid u_2, x_{i,t}] - f_{u_2}^{(1)}(x_{i,t})\right)$$

1. User Graph Estimation
2. Reward & Potential Gain Estimation
3. Arm Selection

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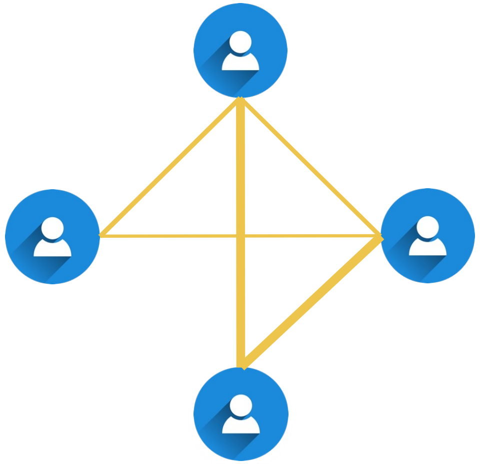
1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

User Exploitation Graph Estimation

User Preference (expected reward) Estimation:

- Estimated by **user exploitation networks** $\left\{f_u^{(1)}\right\}_{u\in\mathcal{U}}$
- Approximating $\mathbb{E}[\boldsymbol{r} \mid \boldsymbol{u}, \boldsymbol{x}]$
- Input: $\boldsymbol{x}$ Label: $\boldsymbol{r}$



User correlations w.r.t. the expected reward (**Exploitation Graph**)

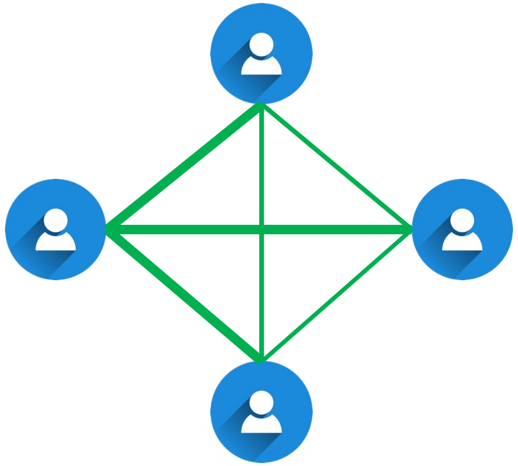
- **User Exploitation Graph Estimation:**
  - ☐ Given arm $\boldsymbol{x}_{i,t}$, **estimated** user exploitation graph $\mathcal{G}_{i,t}^{(1)} = (\mathcal{U}, E, W_{i,t}^{(1)})$
    - $W_{i,t}^{(1)}$: set of estimated edge weights
  - ☐ For users $u_1, u_2 \in \mathcal{U}$, estimated edge weight
    - ○ $w_{i,t}^{(1)}(u_1, u_2) = \boldsymbol{\Psi}^{(1)}\left(\underbrace{f_{u_1}^{(1)}(\boldsymbol{x}_{i,t}),\ f_{u_2}^{(1)}(\boldsymbol{x}_{i,t})}_{\text{Estimated User Preference}}\right)$

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

User Exploration Graph Estimation

Potential Gain:

- Estimated by user **exploration** networks $\{f_u^{(2)}\}_{u \in \mathcal{U}}$
  - **Input**: $\nabla f_u^{(1)}(\boldsymbol{x})$ -- the **gradients** of $f_u^{(1)}$.
  - **Label**: $\boldsymbol{r_u} - \boldsymbol{f_u^{(1)}}(\boldsymbol{x})$.



User correlations w.r.t. the Potential Gain (**Exploration Graph**)

➢ User Exploration Graph Estimation:

☐ Given arm $\boldsymbol{x}_{i,t}$, **estimated** user exploration graph

$$\mathcal{G}_{i,t}^{(2)} = (\mathcal{U}, E, \underbrace{W_{i,t}^{(2)}}_{\text{Edge weight estimations}})$$

☐ For users $u_1, u_2 \in \mathcal{U}$, estimated edge weight

- ○ $w_{i,t}^{(2)}(u_1, u_2) =$

$$\boldsymbol{\psi}^{(2)}\left(\underbrace{f_{u_1}^{(2)}(\nabla f_{u_1}^{(1)}(\boldsymbol{x}_{i,t})),}_{\boxed{\text{Estimated Potential Gain}}}\ f_{u_2}^{(2)}(\nabla f_{u_2}^{(1)}(\boldsymbol{x}_{i,t}))\right)$$

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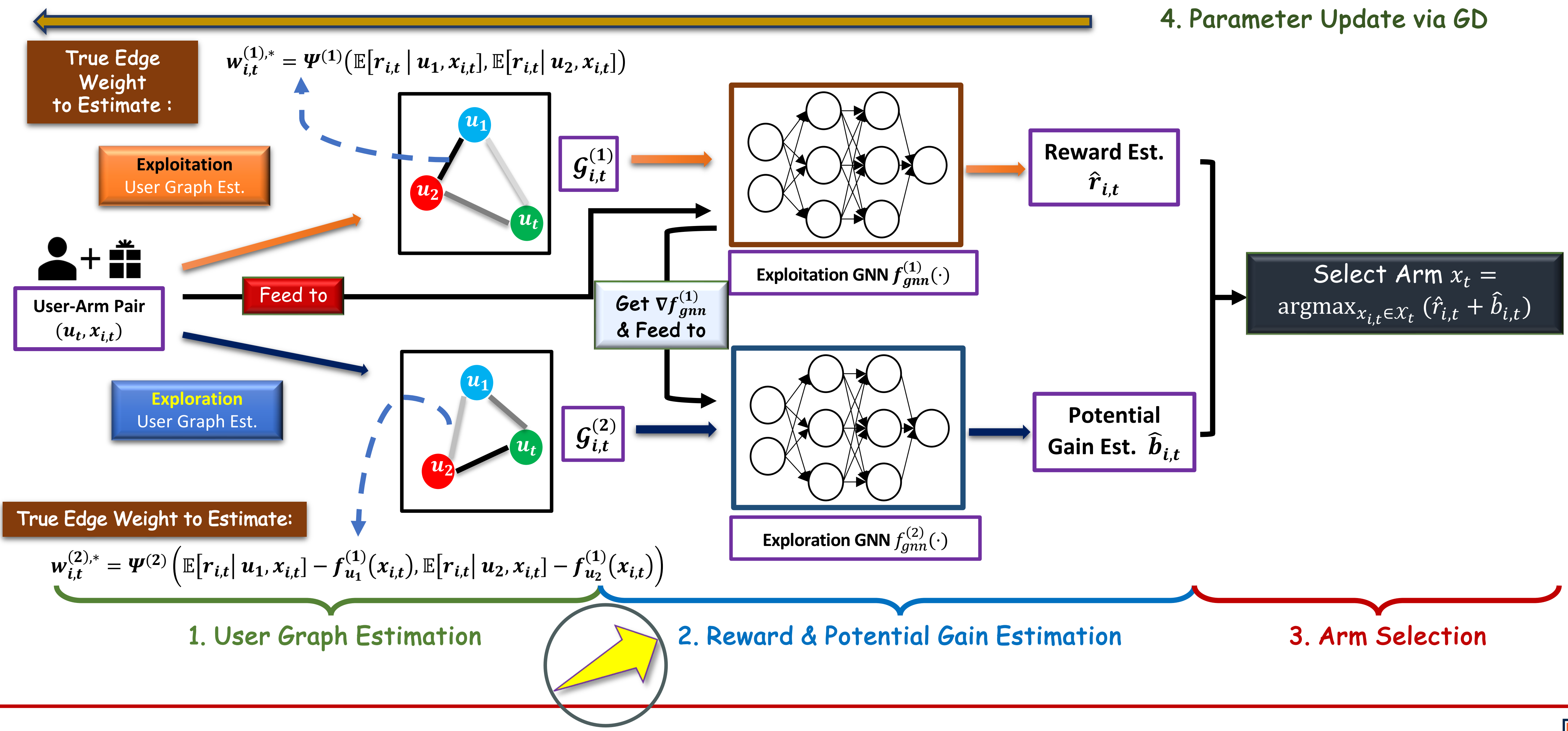
1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.
2. Yikun Ban, et al. EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits. In ICLR 2022.

GNB: Framework Overview



4. Parameter Update via GD

True Edge Weight to Estimate :

$w_{i,t}^{(1),*} = \Psi^{(1)}(\mathbb{E}[r_{i,t} \mid u_1, x_{i,t}], \mathbb{E}[r_{i,t} \mid u_2, x_{i,t}])$

Exploitation User Graph Est.

u_1 u_2 u_t

$\mathcal{G}_{i,t}^{(1)}$

Exploitation GNN $f_{gnn}^{(1)}(\cdot)$

Reward Est. $\hat{r}_{i,t}$

User-Arm Pair $(u_t, x_{i,t})$

Feed to

Get $\nabla f_{gnn}^{(1)}$ & Feed to

Select Arm $x_t = \text{argmax}_{x_{i,t} \in \mathcal{X}_t} (\hat{r}_{i,t} + \hat{b}_{i,t})$

Exploration User Graph Est.

u_1 u_2 u_t

$\mathcal{G}_{i,t}^{(2)}$

Exploration GNN $f_{gnn}^{(2)}(\cdot)$

Potential Gain Est. $\hat{b}_{i,t}$

True Edge Weight to Estimate:

$w_{i,t}^{(2),*} = \Psi^{(2)}\left(\mathbb{E}[r_{i,t} \mid u_1, x_{i,t}] - f_{u_1}^{(1)}(x_{i,t}), \mathbb{E}[r_{i,t} \mid u_2, x_{i,t}] - f_{u_2}^{(1)}(x_{i,t})\right)$

1. User Graph Estimation

2. Reward & Potential Gain Estimation

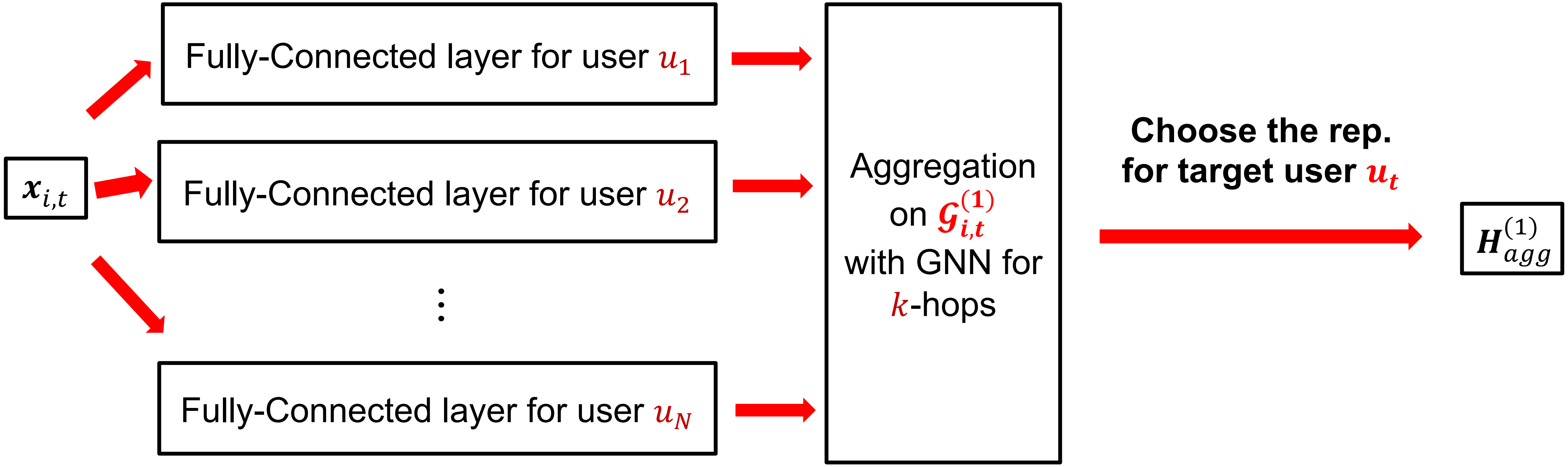
3. Arm Selection

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Aggregation on User Exploitation Graph

- ☐ **For each arm $x_{i,t} \in \mathcal{X}_t$, reward estimation with estimated user exploitation graph $\mathcal{G}_{i,t}^{(1)}$.**

- ➢ **Given target user u_t, obtain User-specific Arm Representation H_{agg}:**



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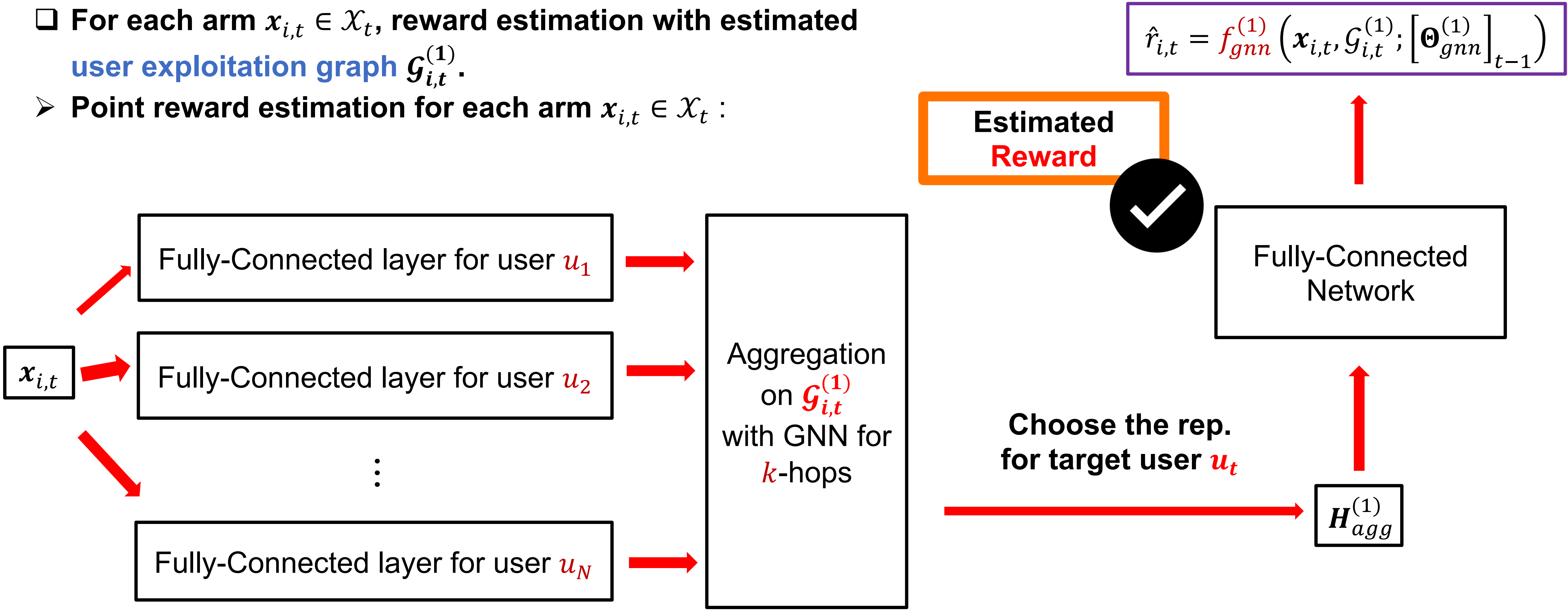
1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Arm Reward Estimation

- ☐ **For each arm $\boldsymbol{x}_{i,t} \in \mathcal{X}_t$, reward estimation with estimated user exploitation graph $\mathcal{G}_{i,t}^{(1)}$.**
- ➢ **Point reward estimation for each arm $\boldsymbol{x}_{i,t} \in \mathcal{X}_t$:**



$$\hat{r}_{i,t} = f_{gnn}^{(1)}\left(\boldsymbol{x}_{i,t}, \mathcal{G}_{i,t}^{(1)}; \left[\boldsymbol{\Theta}_{gnn}^{(1)}\right]_{t-1}\right)$$

Estimated Reward ✓

$\boldsymbol{x}_{i,t}$

Fully-Connected layer for user u_1

Fully-Connected layer for user u_2

⋮

Fully-Connected layer for user u_N

Aggregation on $\mathcal{G}_{i,t}^{(1)}$ with GNN for k-hops

Choose the rep. for target user u_t

$\boldsymbol{H}_{agg}^{(1)}$

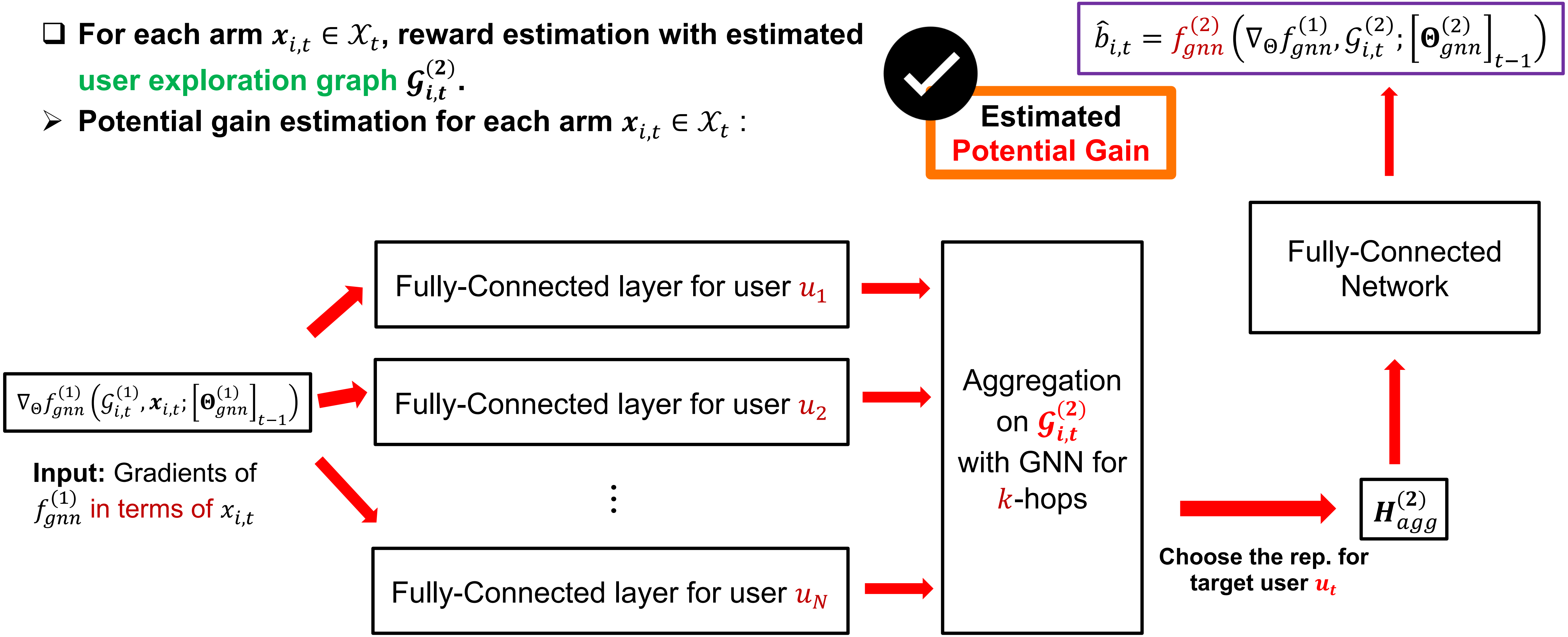
Fully-Connected Network

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Potential Gain Estimation

❑ **For each arm $\boldsymbol{x}_{i,t} \in \mathcal{X}_t$, reward estimation with estimated user exploration graph $\boldsymbol{\mathcal{G}}_{i,t}^{(2)}$.**

➢ **Potential gain estimation for each arm $\boldsymbol{x}_{i,t} \in \mathcal{X}_t$:**



$$\hat{b}_{i,t} = f_{gnn}^{(2)}\left(\nabla_\Theta f_{gnn}^{(1)}, \mathcal{G}_{i,t}^{(2)}; \left[\boldsymbol{\Theta}_{gnn}^{(2)}\right]_{t-1}\right)$$

✓ **Estimated Potential Gain**

$$\nabla_\Theta f_{gnn}^{(1)}\left(\mathcal{G}_{i,t}^{(1)}, \boldsymbol{x}_{i,t}; \left[\boldsymbol{\Theta}_{gnn}^{(1)}\right]_{t-1}\right)$$

Input: Gradients of $f_{gnn}^{(1)}$ in terms of $x_{i,t}$

Fully-Connected layer for user u_1

Fully-Connected layer for user u_2

⋮

Fully-Connected layer for user u_N

Aggregation on $\boldsymbol{\mathcal{G}}_{i,t}^{(2)}$ with GNN for k-hops

Choose the rep. for target user u_t

$\boldsymbol{H}_{agg}^{(2)}$

Fully-Connected Network

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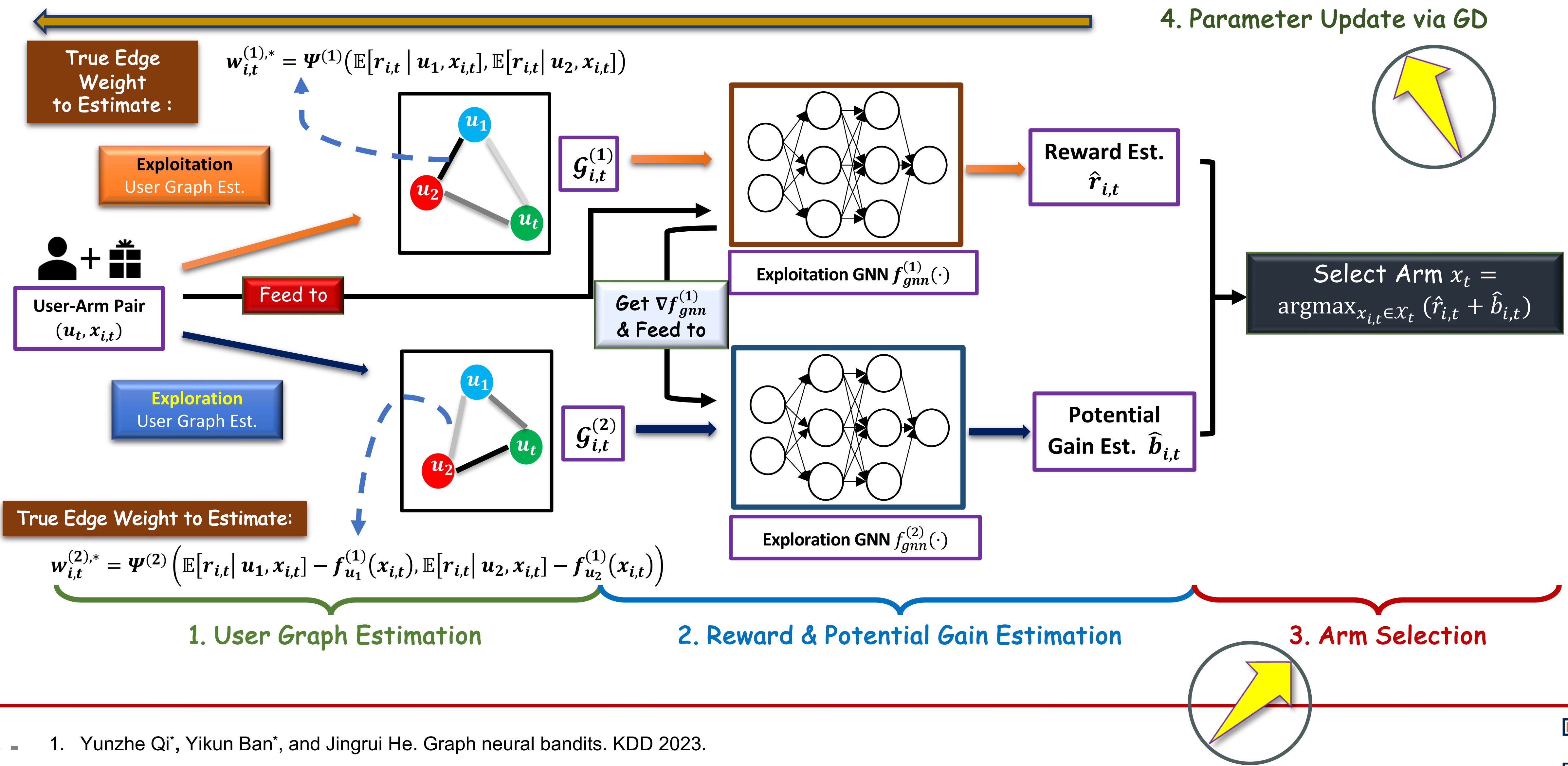

4. Parameter Update via GD

True Edge Weight to Estimate :

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Exploitation User Graph Est.

User-Arm Pair $(u_t, x_{i,t})$

Feed to

Get $\nabla f_{gnn}^{(1)}$ & Feed to

Exploration User Graph Est.

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$\mathcal{G}_{i,t}^{(1)}$

$\mathcal{G}_{i,t}^{(2)}$

Exploitation GNN $f_{gnn}^{(1)}(\cdot)$

Exploration GNN $f_{gnn}^{(2)}(\cdot)$

Reward Est. $\hat{r}_{i,t}$

Potential Gain Est. $\hat{b}_{i,t}$

Select Arm $x_t = \text{argmax}_{x_{i,t} \in \mathcal{X}_t} (\hat{r}_{i,t} + \hat{b}_{i,t})$

1. User Graph Estimation

2. Reward & Potential Gain Estimation

3. Arm Selection

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Arm Selection & Training

GNN + User Exploitation Graph

GNN + User Exploration Graph

- **Arm Selection Strategy:**
  - ☐ Select arm $\boldsymbol{x}_t = \mathrm{argmax}_{x_{i,t} \in \mathcal{X}_t} \left(\underbrace{\hat{r}_{i,t}}_{\text{Estimated Reward}} + \underbrace{\hat{b}_{i,t}}_{\text{Estimated Potential Gain}} \right)$.
  - ☐ Receive the corresponding true reward r_t.
- Train the model parameters (User models + GNN models) with **Gradient Descent (GD)**.
- Update user graphs.

---

1. Yunzhe Qi*, Yikun Ban*, and Jingrui He. Graph neural bandits. KDD 2023.

GNB: Theoretical Analysis

- **Pseudo regret for T rounds:**

$$\boldsymbol{R}(\boldsymbol{T}) = \sum_{t=1}^{T} \mathbb{E}[(r_t^* - r_t)]$$

- **Given sufficiently large network width** m (over-parameterization), under **mild assumptions**, with the probability at least $1 - \delta$:

$$R(T) \le \sqrt{T} \cdot \left(O(L\xi_L) \cdot \sqrt{2\log(\frac{Tn \cdot a}{\delta})}\right) + \sqrt{T} \cdot O(L) + O(\xi_L) + O(1).$$

where n is the number of users, a is the number of arms in each round, and T is the number of rounds.

Remarks:

- ☐ Achieves the regret bound of $\mathcal{O}(\sqrt{T\log(nT)}\,)$.
  - Existing works with user clustering need $\boldsymbol{\mathcal{O}}(\sqrt{nT\log(T)})$ for user collaboration modeling.
- ☐ Free of the terms d
  - d (arm context dimension, common in linear bandit works)

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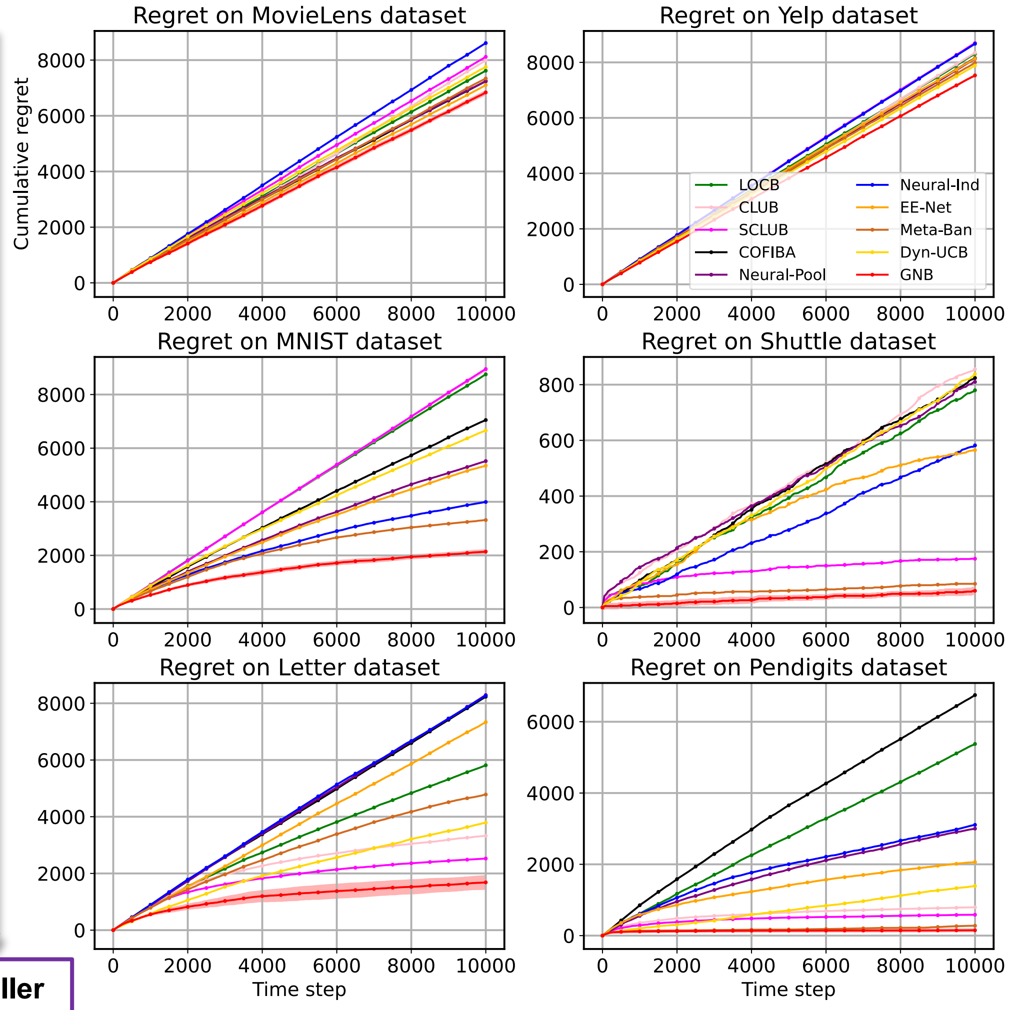
1. Shuai Li, et al. 2019. Improved Algorithm on Online Clustering of Bandits. In IJCAI. 2923–2929.
2. Shuai Li, et al. 2016. Collaborative filtering bandits. In SIGIR. 539–548.
3. Dongruo Zhou, et al. 2020. Neural contextual bandits with ucb-based exploration. In ICML. 11492–11502.
4. Yikun Ban, et al. 2022. Neural Collaborative Filtering Bandits via Meta Learning. arXiv preprint arXiv:2201.13395 (2022).

Experiments: Real Data Sets

- **Experiment settings:**
  - ☐ Under **online recommendation** settings, we evaluate the proposed GNB framework on **six** real data sets with different specifications.
  - ☐ We include **nine** state-of-the-art related algorithms as the baselines, including both linear and neural algorithms.

- **Summary of experiment results:**
  - ☐ **Neural algorithms** generally perform better than **linear ones**, with the representation power of neural networks.
  - ☐ GNB can generally achieve the **best performance** against the strong baselines.



Smaller = Better

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1. Shuai Li, et al. 2019. Improved Algorithm on Online Clustering of Bandits. In IJCAI. 2923–2929.
2. Shuai Li, et al. 2016. Collaborative filtering bandits. In SIGIR. 539–548.
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5. Trong T Nguyen and Hady W Lauw. 2014. Dynamic clustering of contextual multi-armed bandits. In CIKM. 1959–1962.
6. Claudio Gentile, et al. 2014. Online clustering of bandits. In ICML. 757–765.
7. Yikun Ban and Jingrui He. 2021. Local clustering in contextual multi-armed bandits. 2021. In WWW. 2335–2346.
8. Yikun Ban, et al. 2022. EE-Net: Exploitation-Exploration Neural Networks in Contextual Bandits. In ICLR.

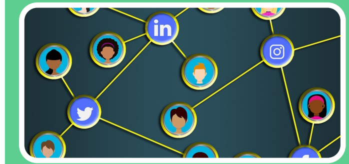
Roadmap

Introduction

- Background & Motivations
- Challenges

Online Clustering of Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits



Graph Bandit Learning with Collaboration

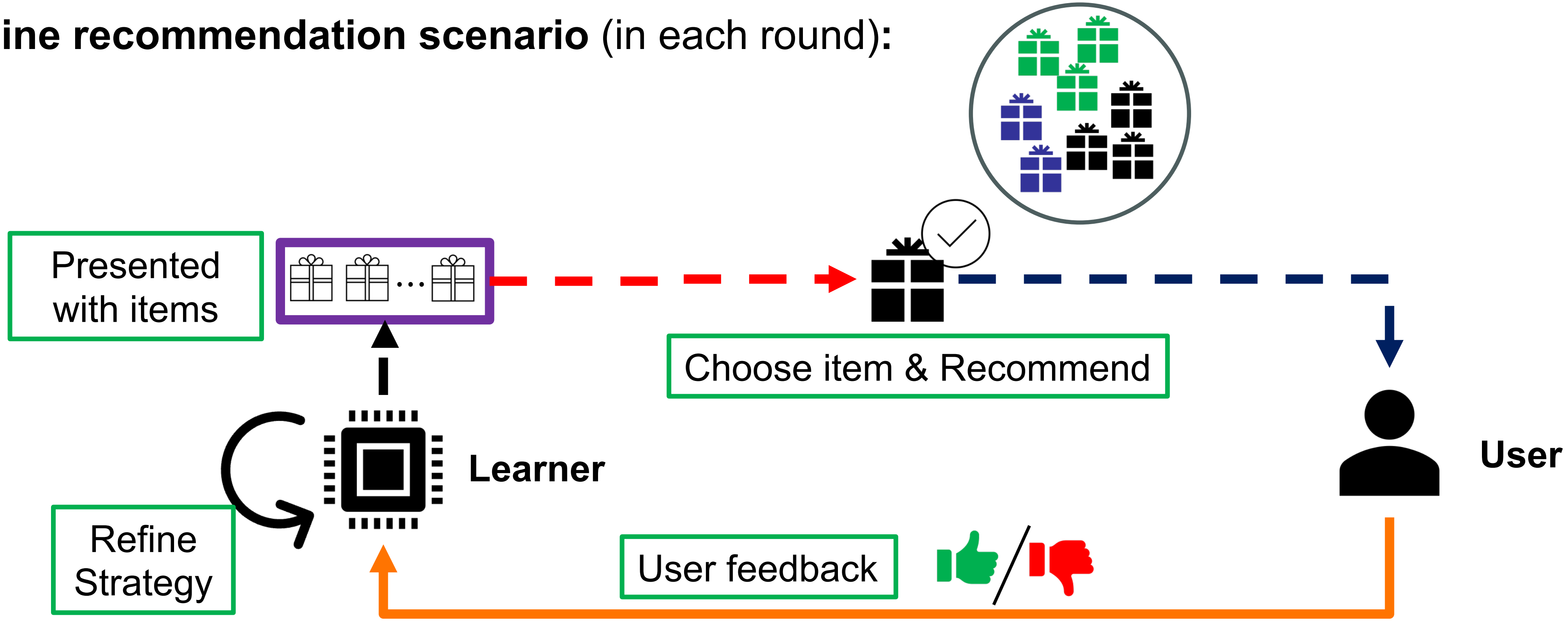
- User side: Graph Neural Bandits
- Arm side: Neural Bandit with Arm Group Graph
- Other Scenarios: Bandit Learning with Graph Feedback & Online Graph Classification with Neural Bandit

Bandits for Combo Recommendation

- Multi-facet Contextual Bandits

Online Recommendation with Arm Group Information

- **Online recommendation scenario** (in each round):



Leverage the available arm group information can help improve recommendation quality.

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1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Arm Group Information

- **The group (category) information for arms (items) is commonly accessible:**
  - ☐ **Media contents:**
    - ○ Music, Movies (grouped by genres)
  - ☐ **Text contents:**
    - ○ Articles (grouped by literary styles)
  - ☐ **E-commerce:**
    - ○ Restaurants (grouped by cuisine types)
  - ☐ Etc.

No existing **MAB** method trying to directly leverage the available arm group information.

1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Formal Problem Definition

- **Arm Groups:**
  - Assume a fixed pool $\mathcal{C}$ of $|\mathcal{C}| = N_c$ **arm groups**.
  - Each **arm group** $c \in \mathcal{C}$ (e.g., movie genre) relates to an arm distribution $\mathcal{D}_c$.
- **For each round** $t \in [T]$ **:**
  - ❑ Receive a set of arms $\mathcal{X}_t$, and the corresponding set of **arm groups** $\mathcal{C}_t \subseteq \mathcal{C}$.
    - $\mathcal{X}_t = \left\{ \boldsymbol{x}_{c,t}^{(i)} \in \mathbb{R}^{d_x}, \ \ (\text{e.g., } 🎞) \right\}_{c \in \mathcal{C}_t,\, i \in [n_{c,t}]}$
    - $\boldsymbol{x}_{c,t}^{(i)} \sim \mathcal{D}_c$
  - ❑ Reward $r_{c,t}^{(i)} = h\left(W^*, x_{c,t}^{(i)}\right) + \epsilon_{c,t}^{(i)}$.
    - Unknown affinity matrix for arm groups: $W^* \in \mathbb{R}^{N_c \times N_c}$
  - ❑ Learner **chooses** arm $x_t \in \mathcal{X}_t$.

- **Objective: Minimizing Pseudo Regret**

$$R(T) = \sum_{t=1}^{T} \mathbb{E}[(r_t^* - r_t)]$$

$$= \sum_{t=1}^{T} |\underbrace{h(W^*, x_t^*)}_{\text{Optimal arm}} - \underbrace{h(W^*, x_t)}_{\text{Chosen arm}}|$$

---

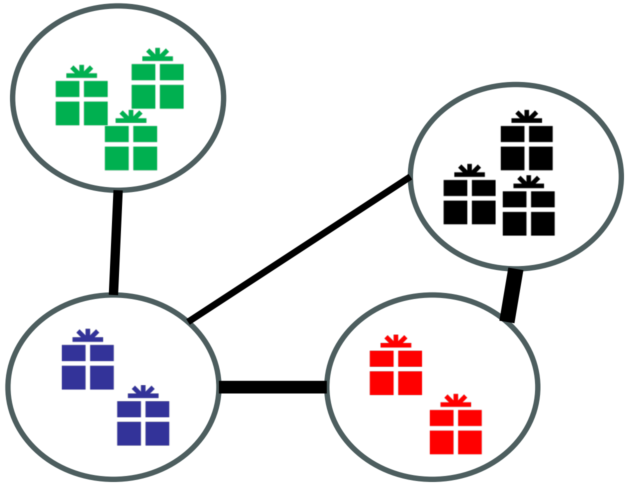
1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Modeling with Arm Group Graph (AGG)

- **Apply Arm Group Graph (AGG) to model arm group correlations:**

- **In round $t \in [T]$:**
  - ☐ Undirected graph $\mathcal{G}_t = (V, E, W_t)$
    - V: set of nodes
      - **Each node is an arm group $c \in \mathcal{C}$,** N_c nodes in total
    - $E = \{e(c, c')\}_{c,c' \in \mathcal{C}}$: set of edges
    - W_t: Set of edge weights
  - ☐ Arm group correlations are modeled by the edge weights from set W_t.

- **True reward**: $r_{c,t}^{(i)} = h\left(\mathcal{G}^*, x_{c,t}^{(i)}\right) + \epsilon_{c,t}^{(i)}$.
  - ○ **Unknown true graph: $\mathcal{G}^*$**
  - ○ Unknown affinity matrix: $W^* \in \mathbb{R}^{N_c \times N_c}$



Arm Group Graph

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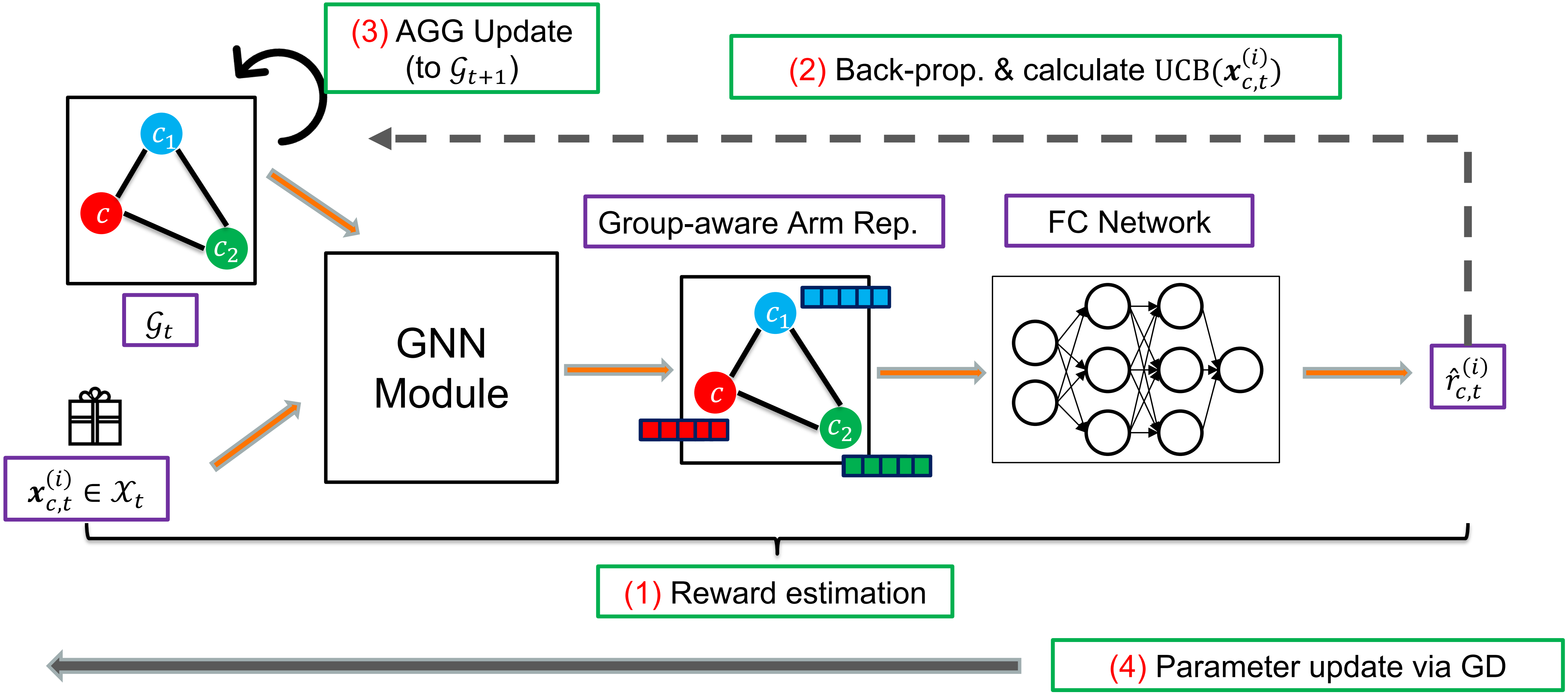
1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Proposed Framework: AGG-UCB



(3) AGG Update (to $\mathcal{G}_{t+1}$)

(2) Back-prop. & calculate $\mathrm{UCB}(\boldsymbol{x}_{c,t}^{(i)})$

$\mathcal{G}_t$

GNN Module

Group-aware Arm Rep.

FC Network

$\boldsymbol{x}_{c,t}^{(i)} \in \mathcal{X}_t$

$\hat{r}_{c,t}^{(i)}$

(1) Reward estimation

(4) Parameter update via GD

1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

AGG-UCB: Arm Group Graph Estimation

- **Recall for Arm Groups:**
  - ○ Assume a fix pool $\mathcal{C}$ of $|\mathcal{C}| = N_c$ arm groups.
  - ○ Each group $c \in \mathcal{C}$ has a context distribution $\mathcal{D}_c$.

➢ **Definition: True edge weights**

❑ For $c,\ c' \in \mathcal{C}$, true edge weight in $\mathcal{G}^*$:

- ○ $w^*(c, c') = \exp(\frac{-\left\|\mathbb{E}_{\boldsymbol{x}\sim\mathcal{D}_c}[\phi(\boldsymbol{x})] - \mathbb{E}_{\boldsymbol{x}'\sim\mathcal{D}_{c'}}[\phi(\boldsymbol{x}')]\right\|^2}{\sigma_s})$
- ○ $\phi(\cdot)$: kernel mapping function

➢ **Arm Group Graph estimation:**

❑ Estimated edge weight in round $t \in [T]$:

- ○ $w_t(c, c') = \exp(\frac{-\|\Psi_t(\mathcal{D}_c) - \Psi_t(\mathcal{D}_{c'})\|^2}{\sigma_s})$
- ○ **Kernel Mean Embedding** [1]**:** $\Psi_t(\mathcal{D}_c)$

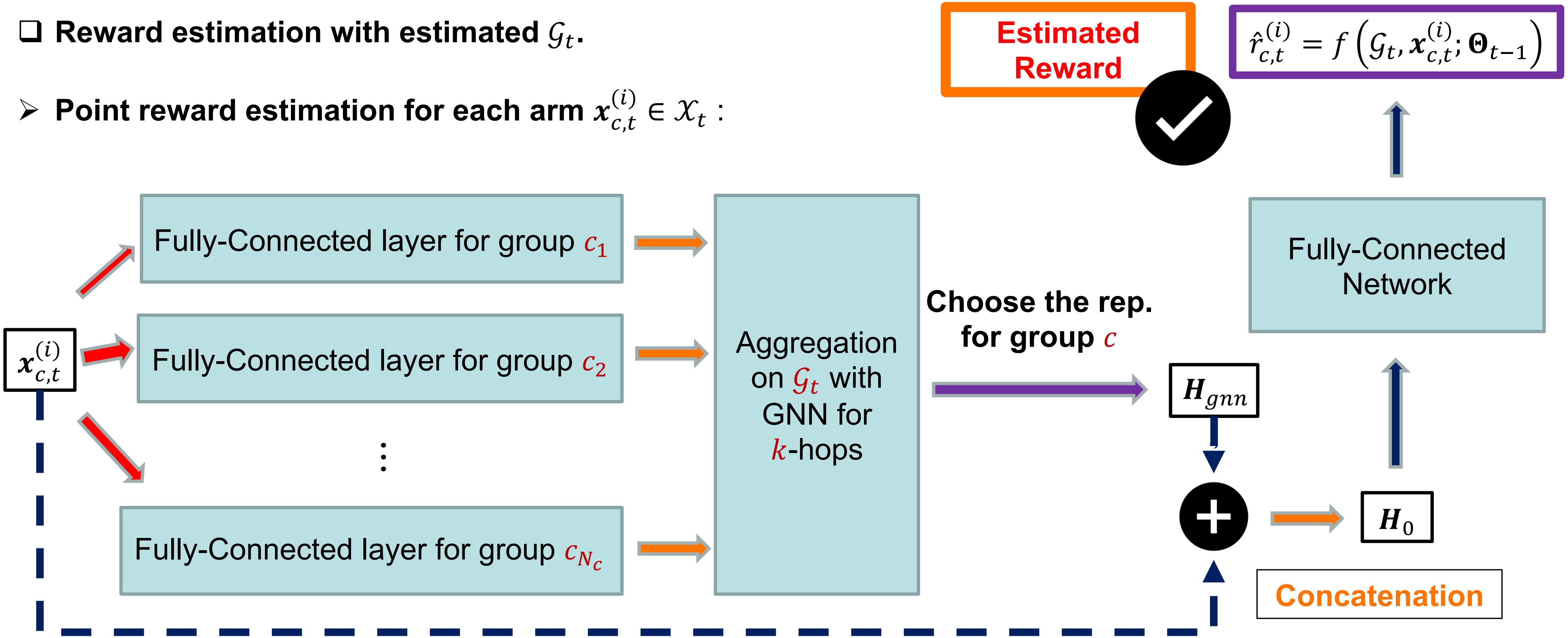
❑ $w_t(c, c') \in W_t$: weight for edge $\mathrm{e}(c, c') \in E$ in graph $\mathcal{G}_t$

---

1. Gilles Blanchard, Gyemin Lee, and Clayton Scott. Generalizing from several related classification tasks to a new unlabeled sample. NeurIPS ‘11.

AGG-UCB: Arm Reward Estimation

- ☐ **Reward estimation with estimated $\mathcal{G}_t$.**
  - ➢ **Point reward estimation for each arm $\boldsymbol{x}_{c,t}^{(i)} \in \mathcal{X}_t$:**



Estimated Reward

$$\hat{r}_{c,t}^{(i)} = f\left(\mathcal{G}_t, \boldsymbol{x}_{c,t}^{(i)}; \boldsymbol{\Theta}_{t-1}\right)$$

$\boldsymbol{x}_{c,t}^{(i)}$

Fully-Connected layer for group c_1

Fully-Connected layer for group c_2

⋮

Fully-Connected layer for group c_{N_c}

Aggregation on $\mathcal{G}_t$ with GNN for k-hops

Choose the rep. for group c

$\boldsymbol{H}_{gnn}$

$\boldsymbol{H}_0$

Concatenation

Fully-Connected Network

1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

AGG-UCB: Arm Selection & Training

- **Exploration with Upper Confidence Bound (UCB):**
  - ☐ The $\mathbf{UCB}(\cdot)$ satisfies :

$$\mathbb{P}\left(\left| \underbrace{f\left(\mathcal{G}_t, \boldsymbol{x}_{c,t}^{(i)}; \boldsymbol{\Theta}_{t-1}\right)}_{\text{Reward Est.}} - \underbrace{h\left(\mathcal{G}^*, \boldsymbol{x}_{c,t}^{(i)}\right)}_{\text{Exp. Reward}} \right| > \mathbf{UCB}\left(\boldsymbol{x}_{c,t}^{(i)}\right)\right) \leq \delta$$

- **Arm Selection Strategy:**
  - ☐ Select arm $\boldsymbol{x}_t = \operatorname{argmax}_{x_{c,t}^{(i)} \in \mathcal{X}_t} \left(\hat{r}_{c,t}^{(i)} + \gamma \cdot \mathbf{UCB}\left(\boldsymbol{x}_{c,t}^{(i)}\right) \right)$
  - ☐ Receive the corresponding true reward r_t

---

1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Theoretical and Empirical Results

- **Theoretical**: Given sufficiently large network width m, with the probability at least $1-\delta$:

$$R(T) \leq 2 \cdot (2B_4\sqrt{T} + 2 - B_4) + 2\sqrt{2\widetilde{d}T\log(1+T/\lambda) + 2T}$$
$$\cdot \left(\sqrt{\lambda}S + \sqrt{1 - 2\log(\delta/2) + (\widetilde{d}\log(1+T/\lambda))}\right)$$

Achieves the regret bound of
$\mathcal{O}(\tilde{d}\sqrt{T\log^2(T)\cdot\log(N_c)})$

- **Empirical**: Leveraging **arm group information with AGG-UCB** can improve good performances.

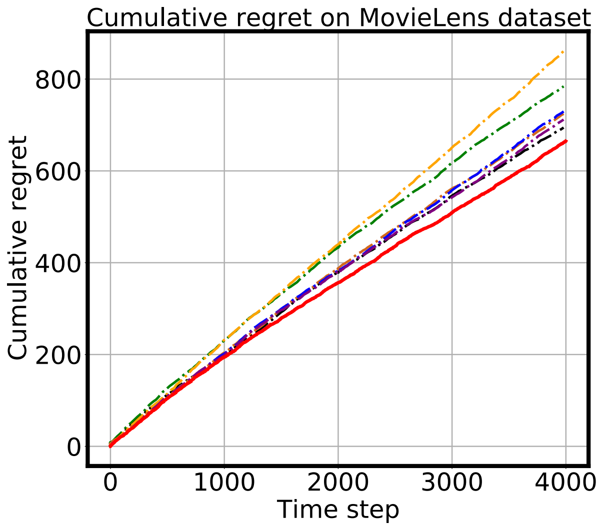


Figure 1: Cumulative regrets on recommendation data sets

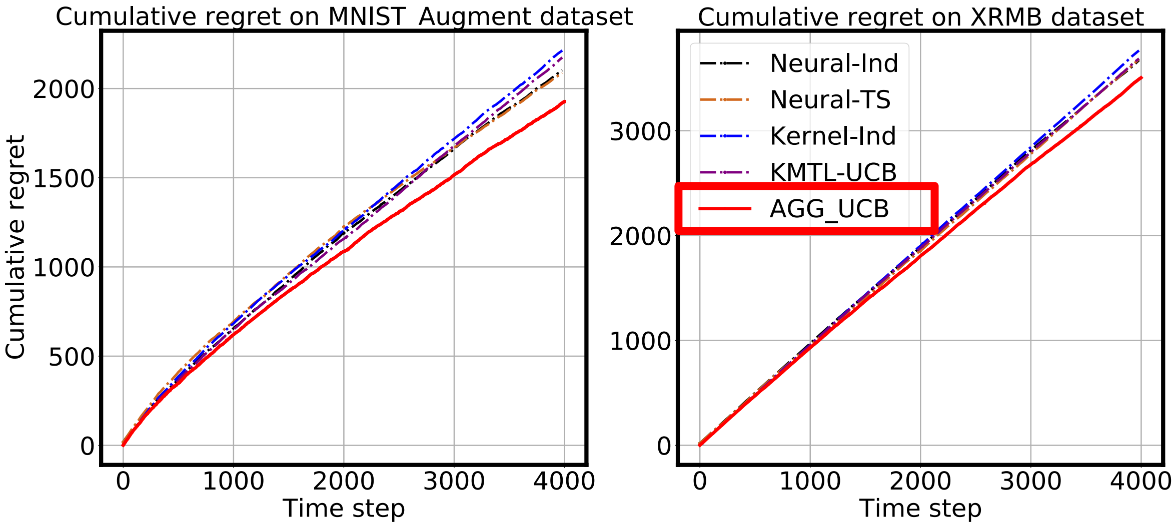


Figure 2: Cumulative regrets on Classification data sets

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1. Yunzhe Qi et. al., Neural bandit with arm group graph. KDD 2022.

Graph Bandit Learning: Other Scenarios

1. **Bandit Learning with Graph Feedback** [1]:

   - ☐ Arms are nodes on a graph $G = (V, E)$. In each round $t \in [T]$, the learner chooses one node $I_t \in V$.
   - ☐ Learner observes **reward for chosen arm** I_t, and **neighbor rewards** (e.g., out-neighbors in a directed graph).
   - ☐ **Objective**: minimizing the cumulative pseudo regret over T rounds.

2. **Optimal Graph Search with Bandit** [2]:

   - ☐ In each round $t \in [T]$, the learner aims to choose **one graph** $G_t \in \mathcal{G}$, from a **fixed** graph domain $\mathcal{G}$. Reward generated by $r_t = h(G_t) + \epsilon_t$.
   - ☐ **Objective**: minimizing the cumulative pseudo regret over T rounds.
   - ☐ **Application example:** material designing, drug search.

---

1. Kong et.al., Simultaneously Learning Stochastic and Adversarial Bandits with General Graph Feedback. ICML 2022.
2. Kassraie et al., Graph Neural Network Bandits. NeurIPS 2022.

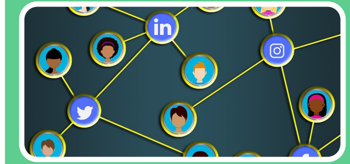
Roadmap

Introduction

- Background & Motivations
- Challenges

Online Clustering of Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits



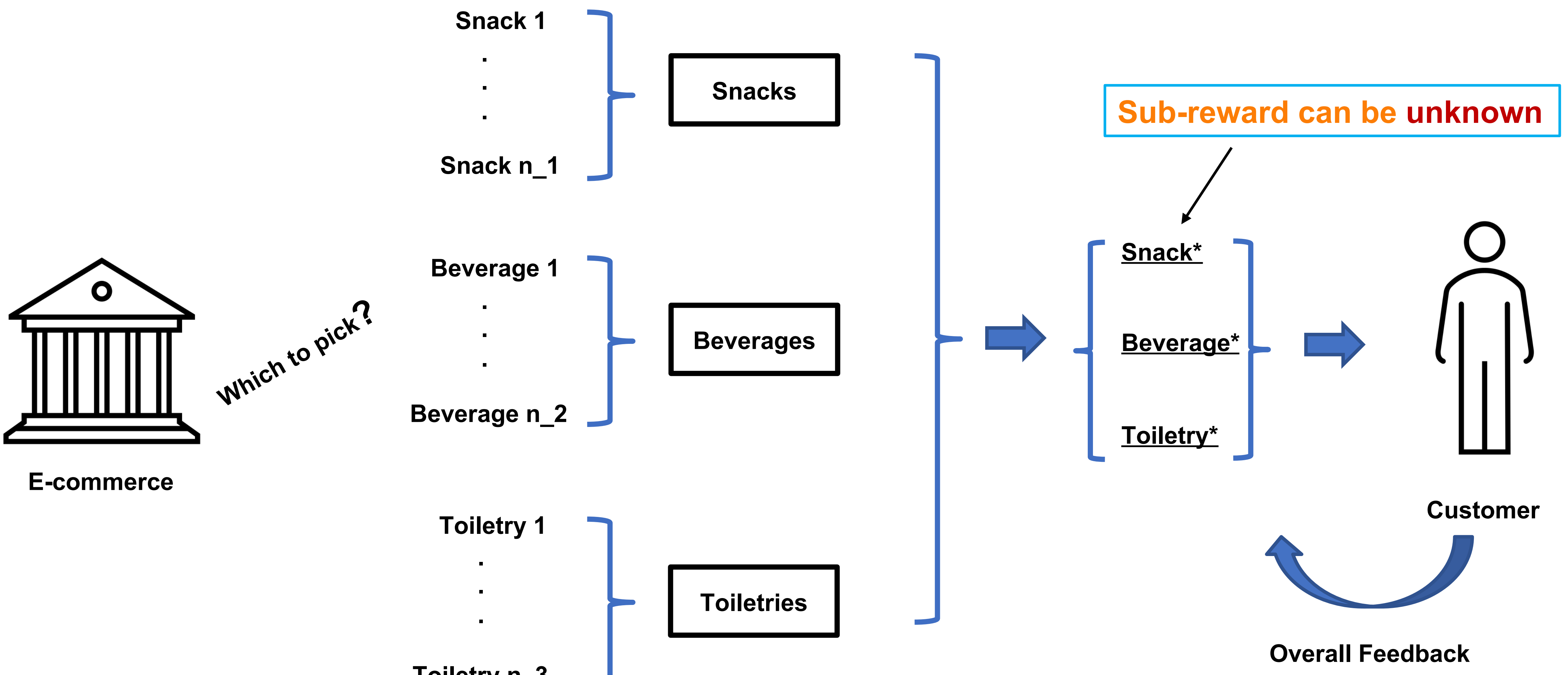
Graph Bandit Learning with Collaboration

- User side: Graph Neural Bandits
- Arm side: Neural Bandit with Arm Group Graph
- Other Scenarios: Bandit Learning with Graph Feedback & Online Graph Classification with Neural Bandit

Bandits for Combo Recommendation

- Multi-facet Contextual Bandits

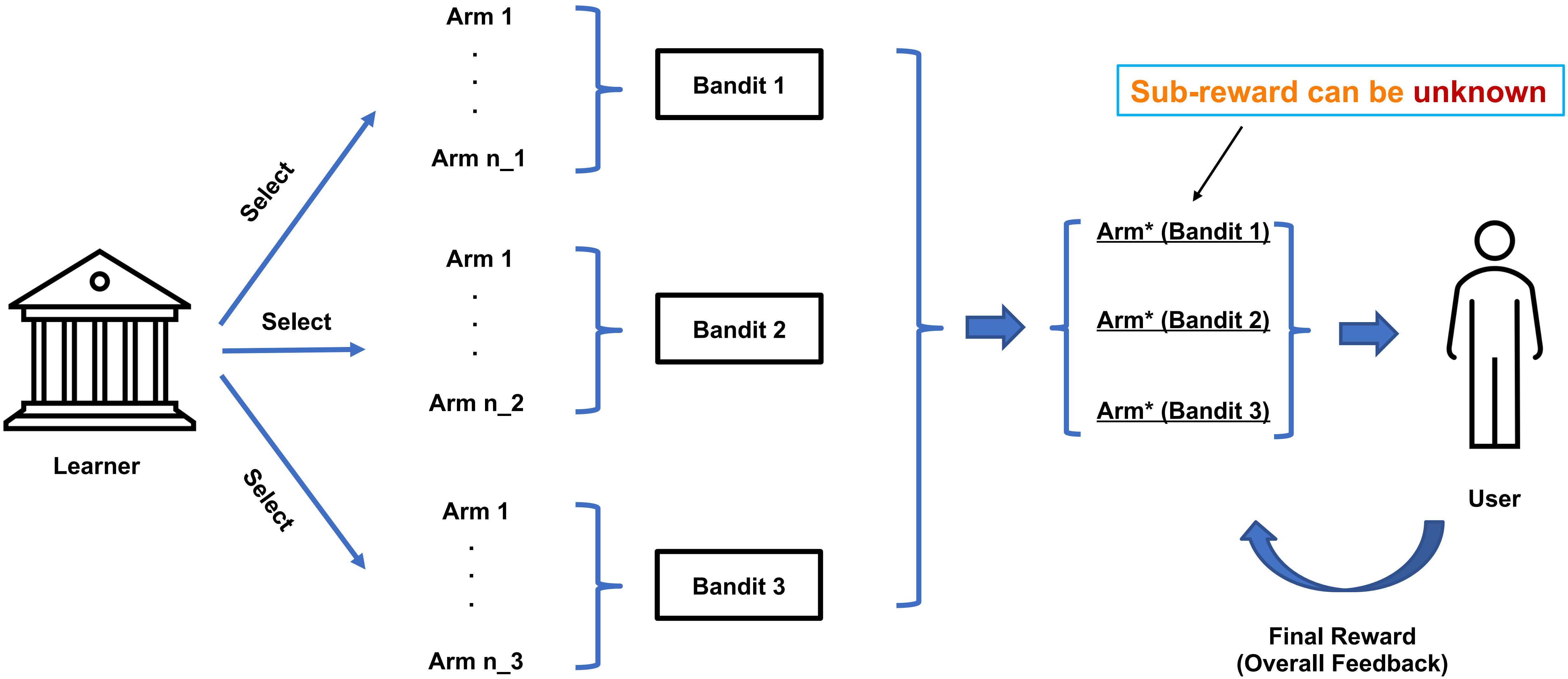
Motivated Case: Promotion Campaign



Sub-reward can be unknown

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

Application: Multi-facet Recommendation with Neural Bandits



1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

Formal Definition of Multi-facet Contextual Bandit : In Round *t*

- **Sub-reward Functions (unknown):**

$$r_t^1 = h_1(x_t^1)$$ (Linear or Non-linear)

$$r_t^2 = h_2(x_t^2)$$

$$\vdots$$

$$r_t^K = h_K(x_t^K)$$

Assumtion1: $h_k(0) = 0, \ \forall \ k$

- **Final Reward Function (unknown):**

$$R_t = H(r_t^1, r_t^2, \ldots, r_t^K) + \boxed{\epsilon_t} \longleftarrow$$ **Noise**

Expectation: $H(X_t) = E[R_t|X_t] = H(r_t^1, r_t^2, \ldots, r_t^K)$

Assumtion2: *H* is $\bar{C}$*- Lipschitz continuous.*

- **Evaluation Measure: Regret**

$$Reg = E\left[\sum_t (R_t^* - R_t)\right]$$

$$= \sum_t [\underbrace{H(X_t^*)}_{\text{Optimal Final Reward}} - \underbrace{H(X_t)}_{\text{Received Final Reward}}]$$

- **Goal: Minimize the regret of T rounds.**

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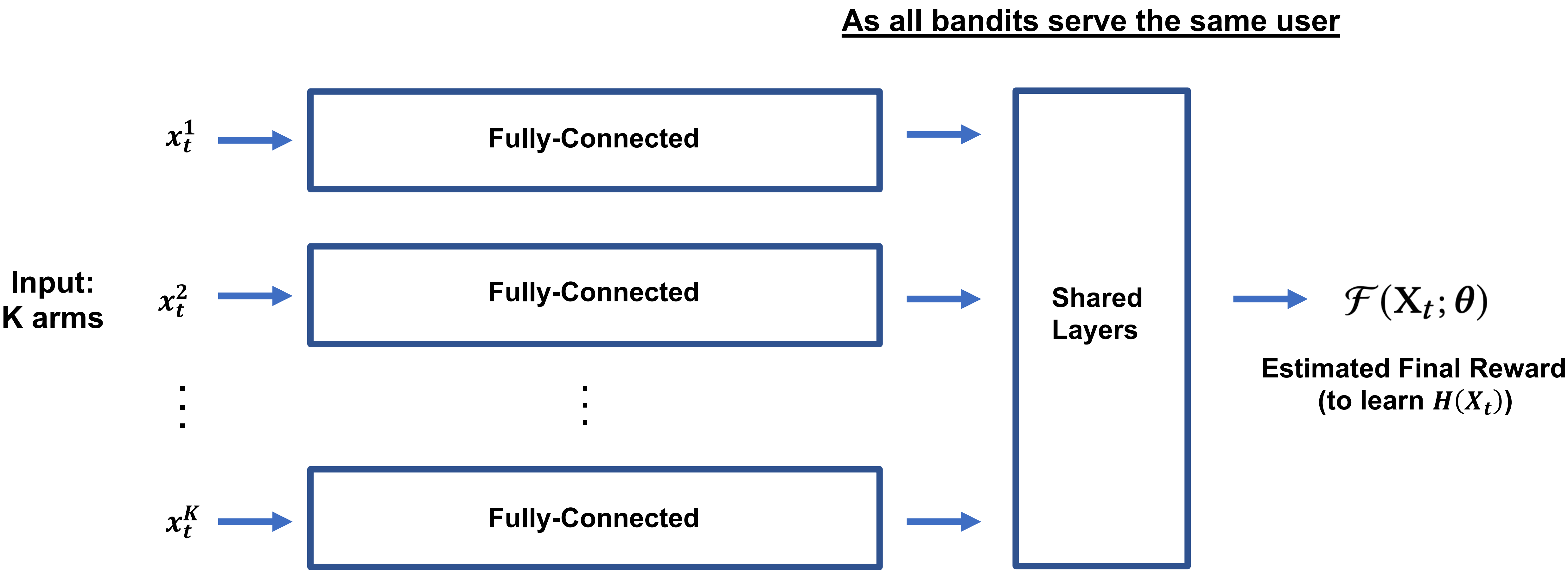
1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

MuFasa: Exploitation (Neural Network Model)



As all bandits serve the same user

Input:
K arms

x_t^1

x_t^2

$\vdots$

x_t^K

Fully-Connected

Fully-Connected

$\vdots$

Fully-Connected

Shared Layers

$\mathcal{F}(\mathbf{X}_t; \boldsymbol{\theta})$

Estimated Final Reward
(to learn $H(X_t)$)

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

MuFasa: Exploration (Upper Confidence Bound)

- **UCB:** $$\mathbb{P}\left(|\mathcal{F}(\mathbf{X}_t;\boldsymbol{\theta}_t) - \mathcal{H}(\mathbf{X}_t)| > \mathrm{UCB}(\mathbf{X}_t)\right) \le \delta,$$

- **K selected arms are determined by:**

$$\mathbf{X}_t = \arg\max_{\mathbf{X}_t' \in \mathbf{S}_t} \left(\mathcal{F}(\mathbf{X}';\boldsymbol{\theta}_t) + \mathrm{UCB}(\mathbf{X}_t')\right).$$

Where

$$\mathbf{S}_t = \{(\mathbf{x}_t^1, \dots, \mathbf{x}_t^k, \dots, \mathbf{x}_t^K) \mid \mathbf{x}_t^k \in \mathbf{X}_t^k, k \in [K]\},$$

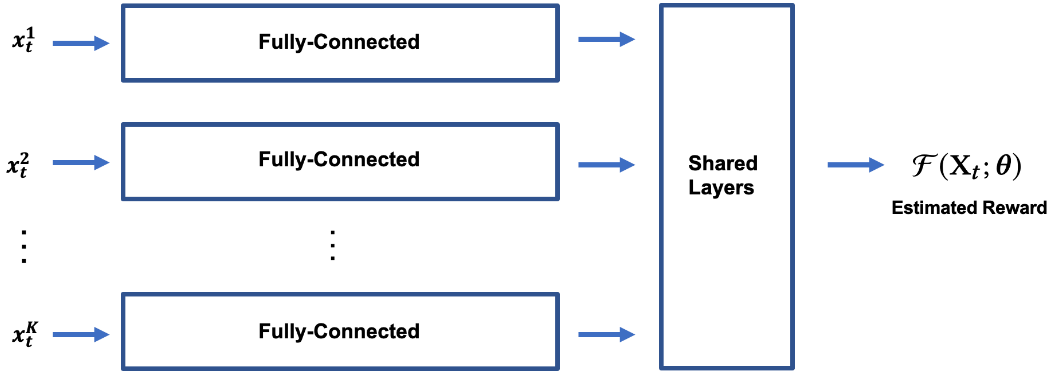
(all possible combinations of K arms)

---

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

MuFasa: Novel Upper Confidence Bound (UCB)

- **With the assembled neural framework (MuFasa):**



- **With probability at least $1-\delta$, the UCB holds**

$$|\mathcal{F}(\mathbf{X}_t;\boldsymbol{\theta}_t) - \mathcal{H}(\mathbf{X}_t)| \le \bar{C}\sum_{k=1}^{K}\mathcal{B}^k + \mathcal{B}^F = UCB(\mathbf{X}_t),\ \ where$$

$$\mathcal{B}^k = \gamma_1\|g_k(\mathbf{x}_t^k;\boldsymbol{\theta}_t^k)/\sqrt{m_1}\|_{\mathbf{A}_t^{k^{-1}}} + \gamma_2(\frac{\delta}{k+1})\|g_k(\mathbf{x}_t^k;\boldsymbol{\theta}_0^k)/\sqrt{m_1}\|_{\mathbf{A}_t^{k'^{-1}}} + \gamma_1\gamma_3 + \gamma_4$$

Error of facet-specific networks

$$\mathcal{B}^F = \gamma_1\|G(\mathbf{f}_t;\boldsymbol{\theta}_t^{\Sigma})/\sqrt{m_2}\|_{\mathbf{A}_t^{F^{-1}}} + \gamma_2(\frac{\delta}{k+1})\|G(\mathbf{f}_t;\boldsymbol{\theta}_0^{\Sigma})/\sqrt{m_2}\|_{\mathbf{A}_t^{F'^{-1}}} + \gamma_1\gamma_3 + \gamma_4$$

Error of shared network

---

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

Regret Analysis

$$\boldsymbol{Reg} = \boldsymbol{E}\left[\sum_{t}(\boldsymbol{R}_t^* - \boldsymbol{R}_t)\right]$$

$$= \sum_{t}[\boldsymbol{H}(\boldsymbol{X}_t^*) - \boldsymbol{H}(\boldsymbol{X}_t)]$$

- ➢ **After T rounds, with probability at least $1 - \delta$,**

$$\boldsymbol{Reg} \leq (\bar{C}K + 1)\sqrt{T}2\sqrt{\widetilde{P}\log(1 + T/\lambda) + 1/\lambda + 1}$$

$$\cdot\left(\sqrt{(\widetilde{P} - 2)\log\left(\frac{(\lambda + T)(1 + K)}{\lambda\delta}\right) + 1/\lambda} + \lambda^{1/2}S + 2\right) + 2(\bar{C}K + 1),$$

- ➢ **Achieve near-optimal regret bound** $\widetilde{O}\left((K + 1)\sqrt{T}\right)$**, same as a single linear bandit** $\widetilde{O}(\sqrt{T})$

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1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

All Sub-rewards Available (Different Final Reward Function)

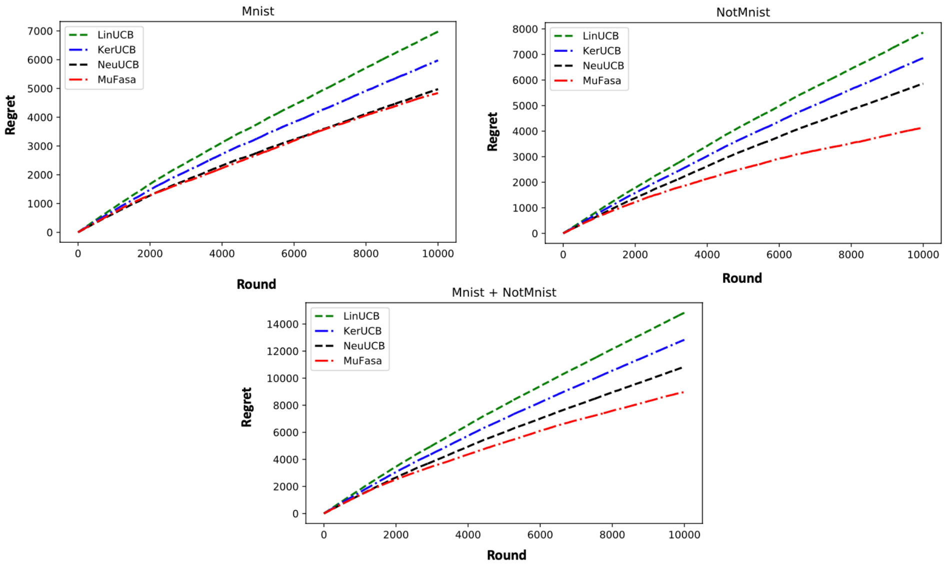


Figure: Regret comparison on Mnist+NotMnist with H_1.

$$H_1(\text{vec}(\mathbf{r}_t)) = r_t^1 + r_t^2$$

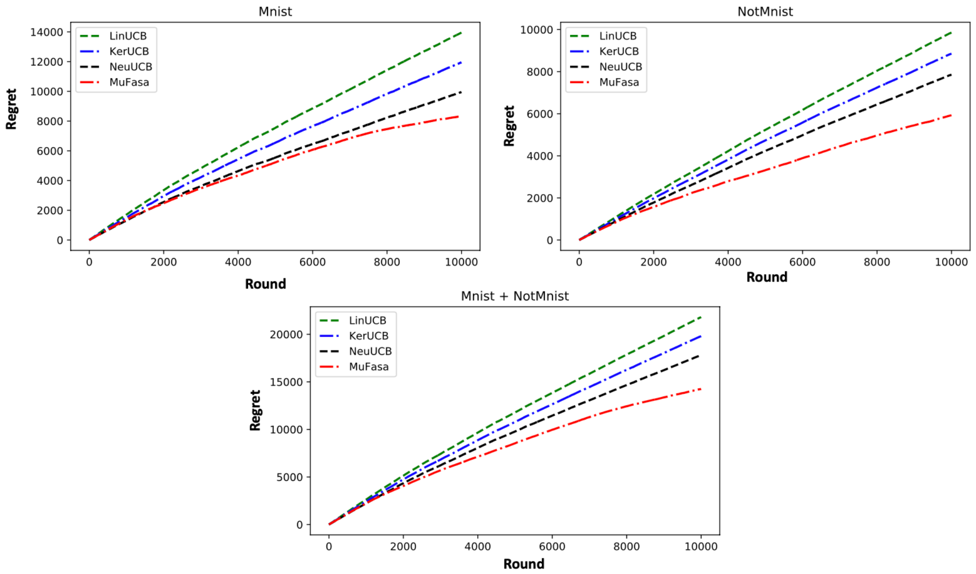


Figure: Regret comparison on Mnist+NotMnist with H_2.

$$H_2(\text{vec}(\mathbf{r}_t)) = 2r_t^1 + r_t^2.$$

Observation:

- **Superiority of MuFasa is slightly higher on H_2, compared to H_1.**

Insights:

- **MuFasa can select arms according to different weights of bandits (Bandit 1 has higher weight in H_2).**

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

Partial Sub-rewards Available

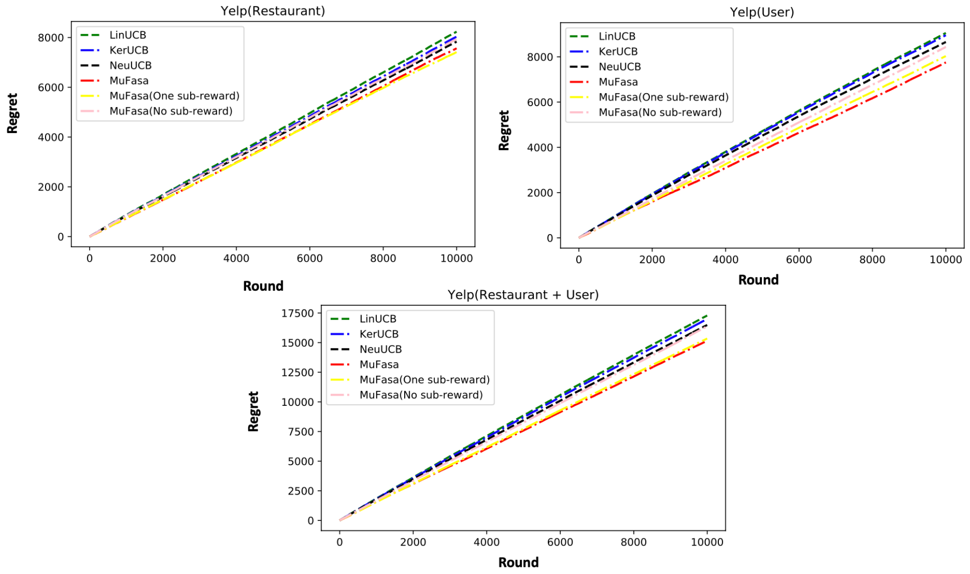


Figure: Regret comparison on Yelp with different reward availability.

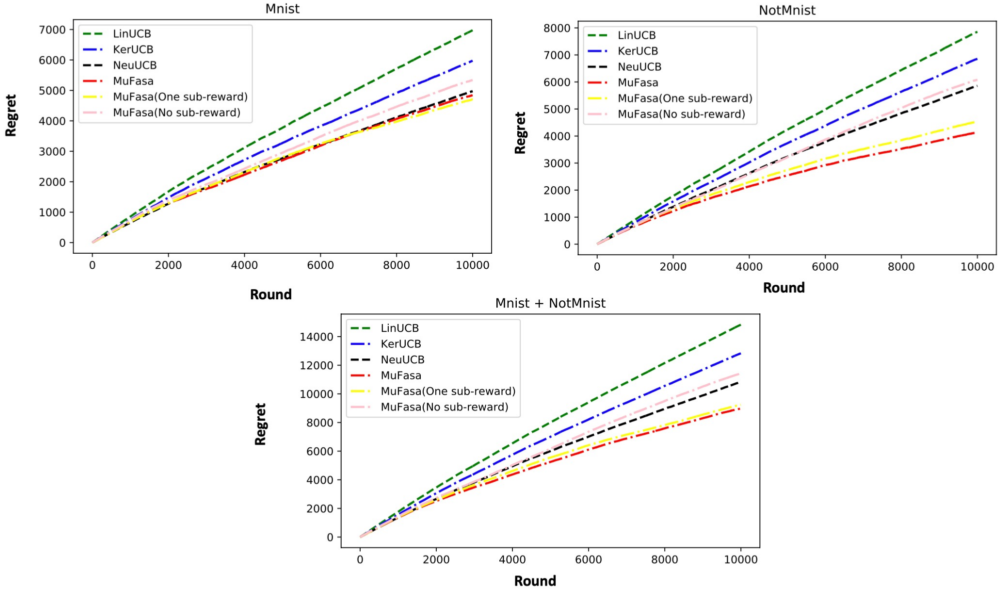


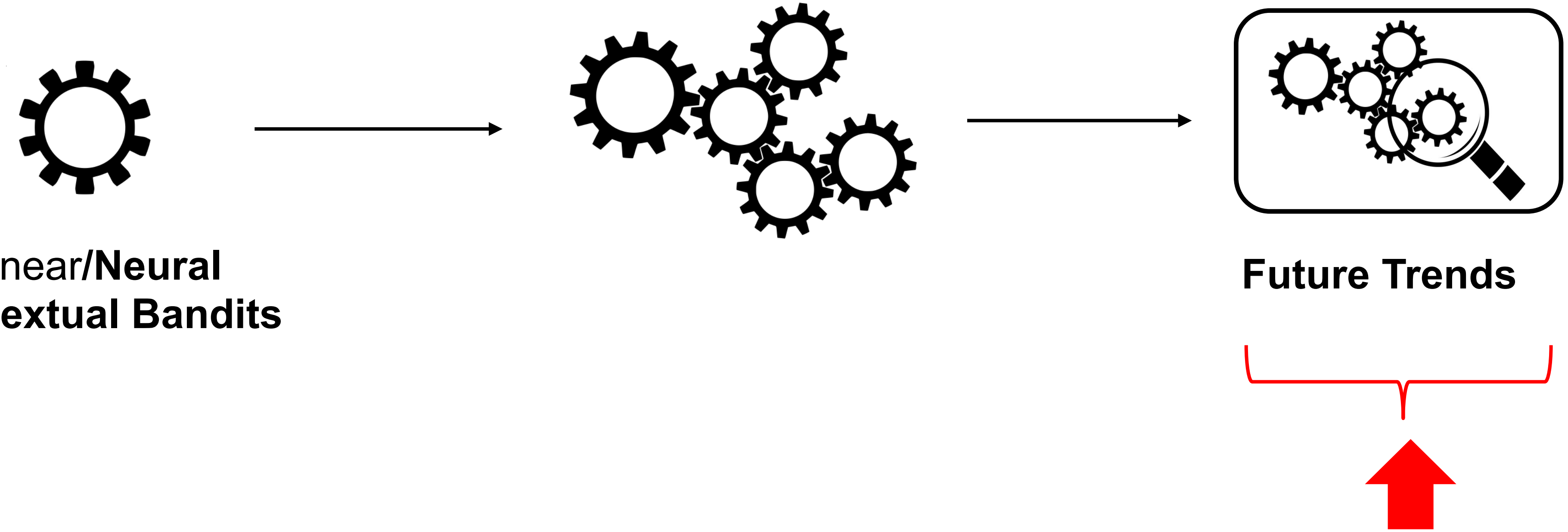
Figure: Regret comparison on Mnist+NotMnist with different reward availability.

Observation:

- With one sub-reward, MuFasa still outperforms all baselines.
- Without any sub-reward, MuFasa's performance is close to the best baseline.

1. Yikun Ban et. al., Multi-facet contextual bandits: A neural network perspective. KDD 2022.

Tutorial Roadmap



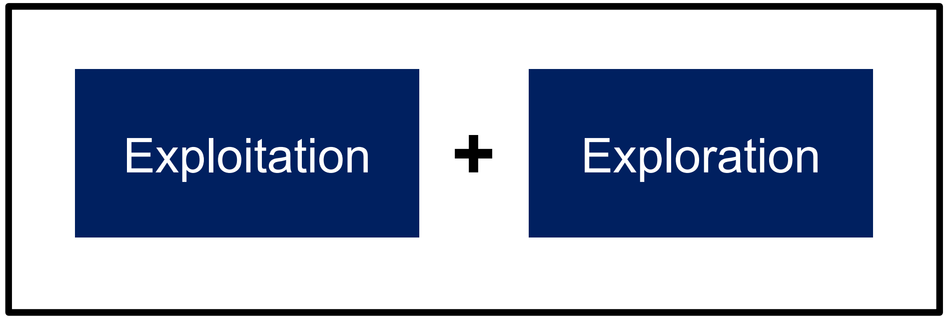
Collaborative Bandits

Linear/**Neural**
Contextual Bandits

Future Trends

Trustworthy Exploration: Transparency

Q: Can we have a **transparent** exploration with clear rationales and explanations?

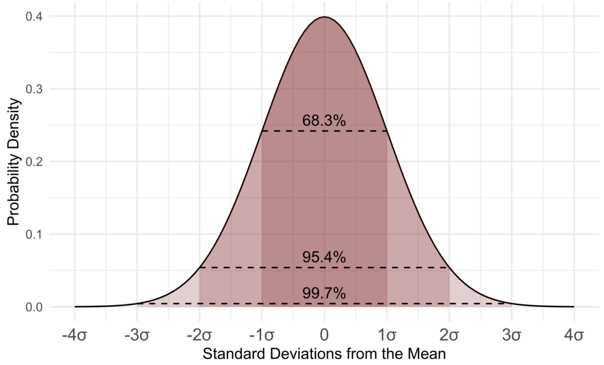

Exploitation + Exploration

Black Box !

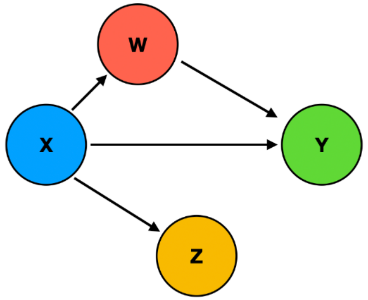
E.g. [Ban et al. ICLR 2022]

- **Challenges:**
  - ❑ More exploration models based on neural networks (**Black Box**).

- **Future Directions:**
  - ❑ Bayesian Bandits/RL.
  - ❑ Causal Bandits/RL.



Statistics



Causal Inference

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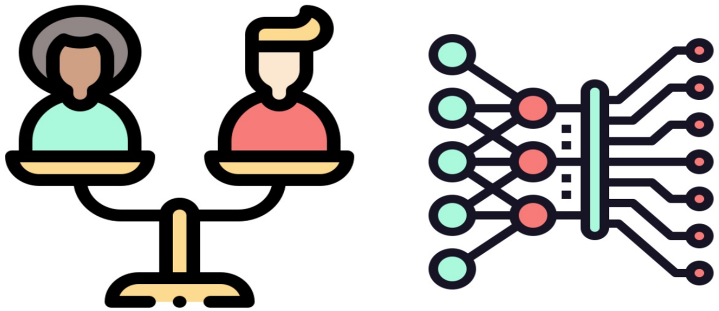
1. Castelvecchi, Davide. "Can we open the black box of AI?." *Nature News* 538.7623 (2016): 20.

Trustworthy Exploration: Fairness

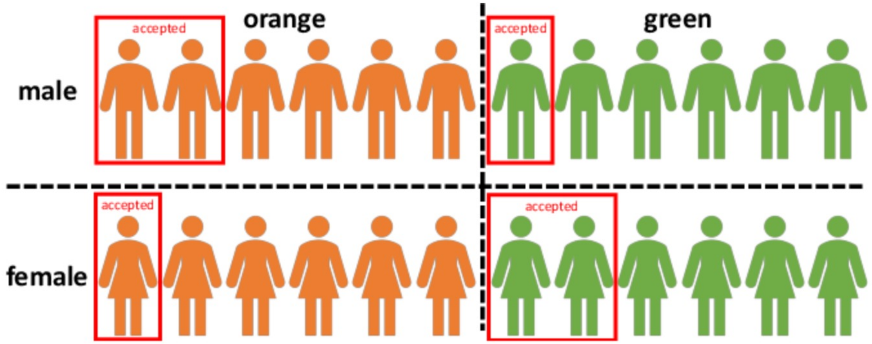
Q: How to ensure **fairness** in the context of exploration?

- **Challenges:**
  - **Non-IID** data.
  - balance required between **exploration power** and **fairness**.

- **Future Directions:**
  - Derive fairness confidence interval for exploration.
  - Fairness Regularization.



Group Fairness



Group Fairness [1]

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1. Joseph, Matthew, et al. "Fairness in learning: Classic and contextual bandits." NeurIPS 2016.
2. Kang, Jian, et al. "Multifair: Multi-group fairness in machine learning." Bigdata 2021.

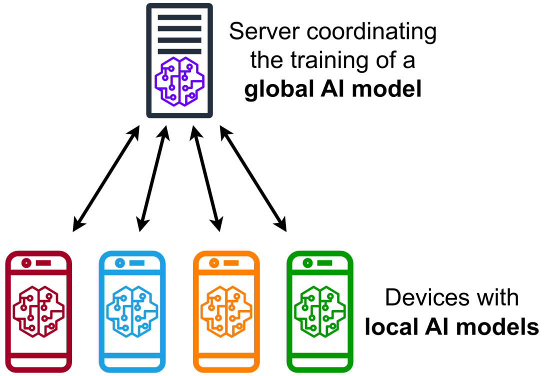
Trustworthy Exploration: Privacy

Q: Can we have an exploration strategy preserving privacy?

- **Challenges:**
  - ❑ Privacy-preserving exploration methods.

- **Future Directions:**
  - ❑ Federated Bandits/RL.

User Privacy



Federated Learning

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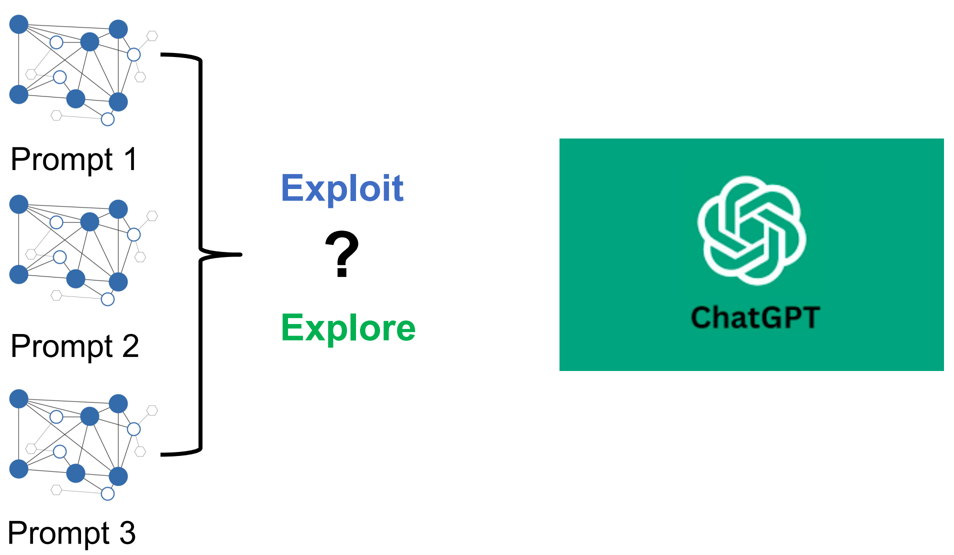
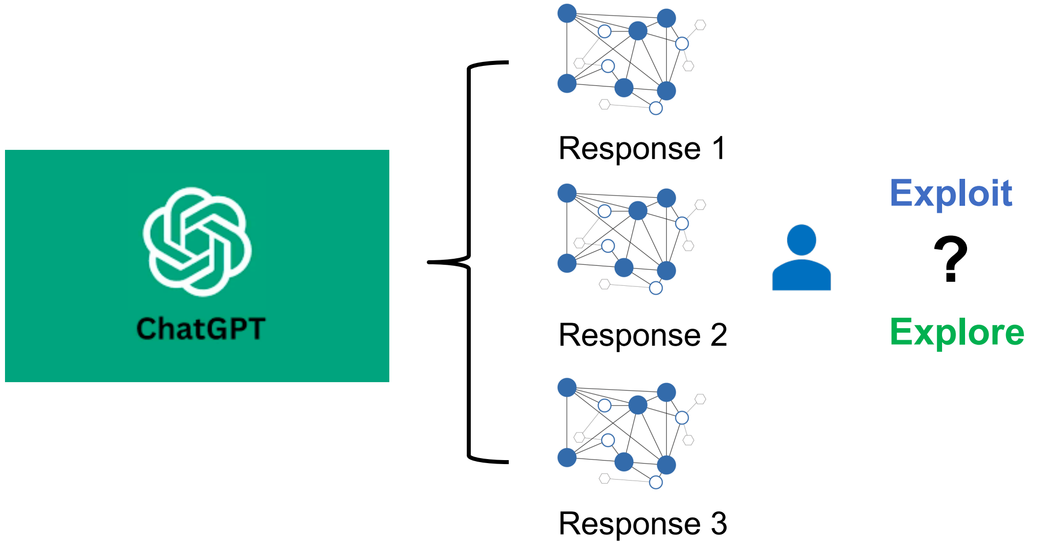
1. Zhang, Chen, et al. "A survey on federated learning." *Knowledge-Based Systems* 216 (2021): 106775.
2. Dai, Zhongxiang, et al. "Federated neural bandits." ICLR 2023.

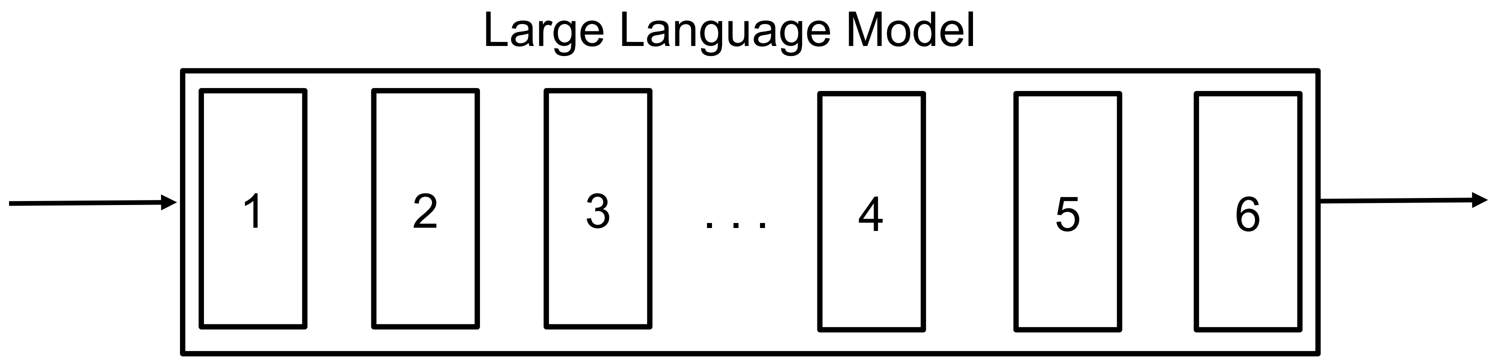
Customized Exploration: Large Language Model



(1) Prompting with Exploration



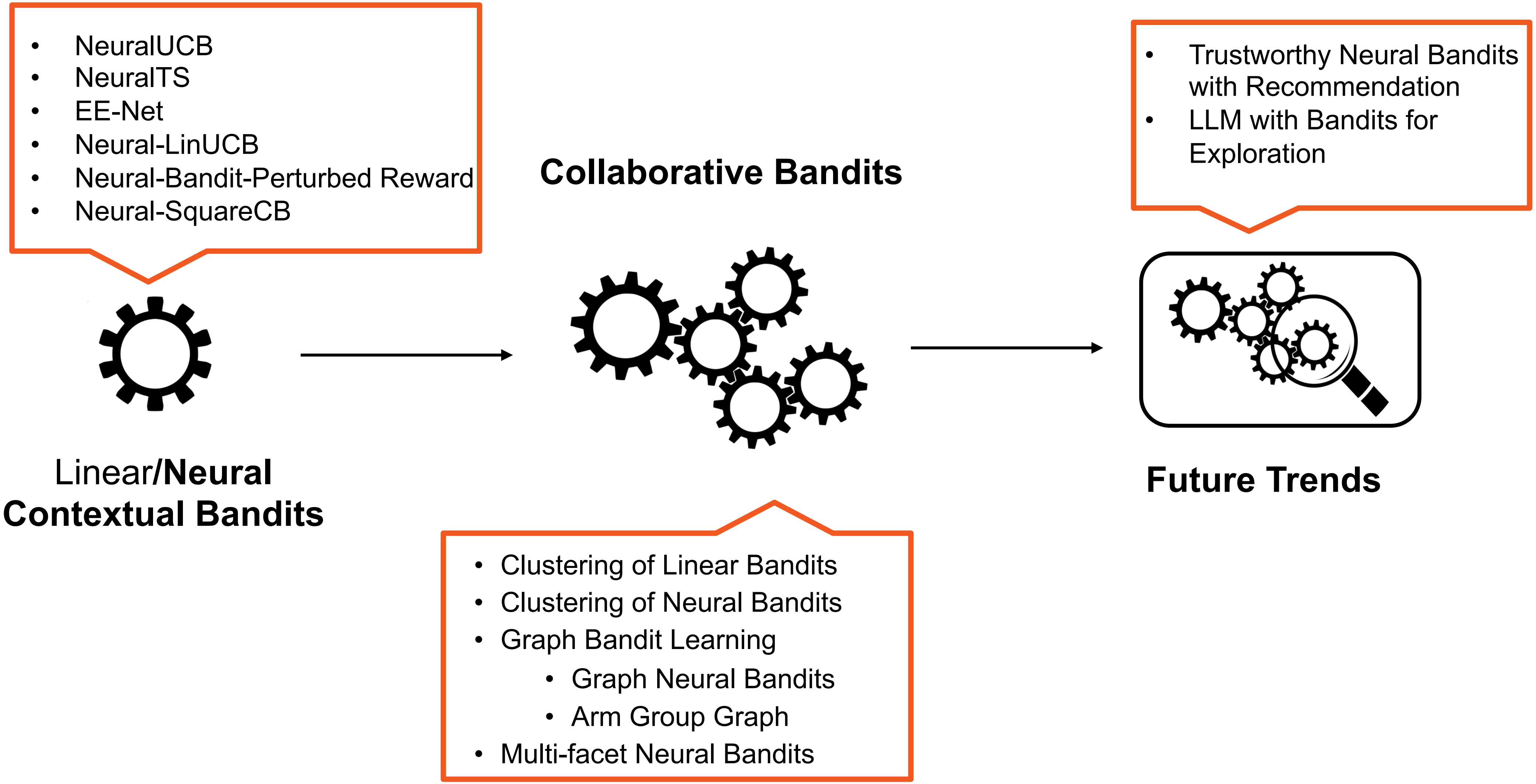
(2) Answering with User-specific Exploration



(3) Fine-tuning with Exploration

1. Lin, Xiaoqiang, et al. "Use your INSTINCT: instruction optimization using neural bandits coupled with transformers." ICML 2024.
2. Chen, Zekai, et al. "Online Personalizing White-box LLMs Generation with Neural Bandits." *arXiv preprint arXiv:2404.16115* (2024).
3. Köpf, Andreas, et al. "Openassistant conversations-democratizing large language model alignment." NeurIPS 2023.
4. Rafailov, Rafael, et al. "Direct preference optimization: Your language model is secretly a reward model." NeurIPS 2023.
5. Zhang, Qingru, et al. "Platon: Pruning large transformer models with upper confidence bound of weight importance." ICML 2022.

Roadmap



- NeuralUCB
- NeuralTS
- EE-Net
- Neural-LinUCB
- Neural-Bandit-Perturbed Reward
- Neural-SquareCB

Linear/**Neural Contextual Bandits**

Collaborative Bandits

- Clustering of Linear Bandits
- Clustering of Neural Bandits
- Graph Bandit Learning
  - Graph Neural Bandits
  - Arm Group Graph
- Multi-facet Neural Bandits

- Trustworthy Neural Bandits with Recommendation
- LLM with Bandits for Exploration

Future Trends

Neural Contextual Bandits for Personalized Recommendation

Yikun Ban

Yunzhe Qi

Jingrui He

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Time: 9:00 AM – 12:30 PM, 13 May 2024

Location: Virgo 1, Resorts World Sentosa Convention Centre, Singapore

Website: www.banyikun.com/wwwtutorial/